Homework 5, Due Monday, Oct 21

This homework must be done individually. Remember to follow Math department's guidelines for homework. Please write your solutions neatly. Typesetting in LaTeX is appreciated and encouraged. **Always show your work and justify your answers.**

---

1. Let U be the subgroup of $\langle \mathbb{C}^*, \cdot \rangle$ consisting of all complex numbers of modulus (absolute value) 1. Give a geometric description of the cosets of U.
2. Suppose that a group G contains elements of orders 1,2,3,4,5,6,7. What is the smallest possible order of G?
3. Suppose H and K are subgroups of a group G. Given that $|H| = 24$ and $|K| = 35$, determine $H \cap K$.
4. Suppose G is a finite group of order p^2, where p is a prime number. Show that either G is cyclic, or $g^p = e$ for all $g \in G$.
5. Let G be a finite group and p be a prime number. Assume that G contains some elements of order p. Show that the number of elements of order p in G is a multiple of $p - 1$.