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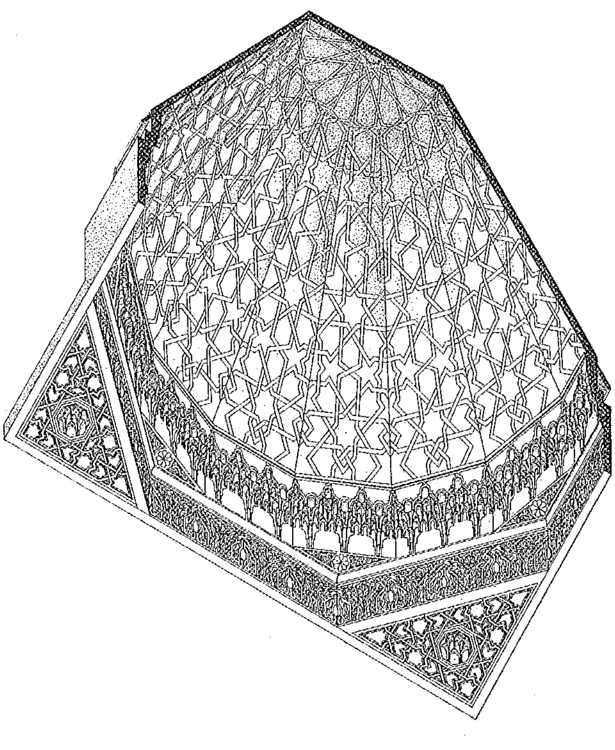
METHODS OF CONSTRUCTING GEOMETRIC ORNAMENTAL SYSTEMS IN THE CUPOLA OF THE ALHAMBRA

In 1981 I was given the task of measuring and drawing the fourteenth-century wooden cupola from the Alhambra which is preserved in the Museum of Islamic Art in West Berlin. While doing this work, I noted three geometric ornaments that are characteristic of Islamic art wherever it is found (fig. 1). The first covers the inner surface of the sixteen sides of the cupola; the second is used to decorate the four carved triangles that fill the octagonal base of the cupola up to a full quadrant. The third is represented by the projection of an inner stalactite cornice on a horizontal plane; it is not visible except by drawing its theoretical graphic representation.

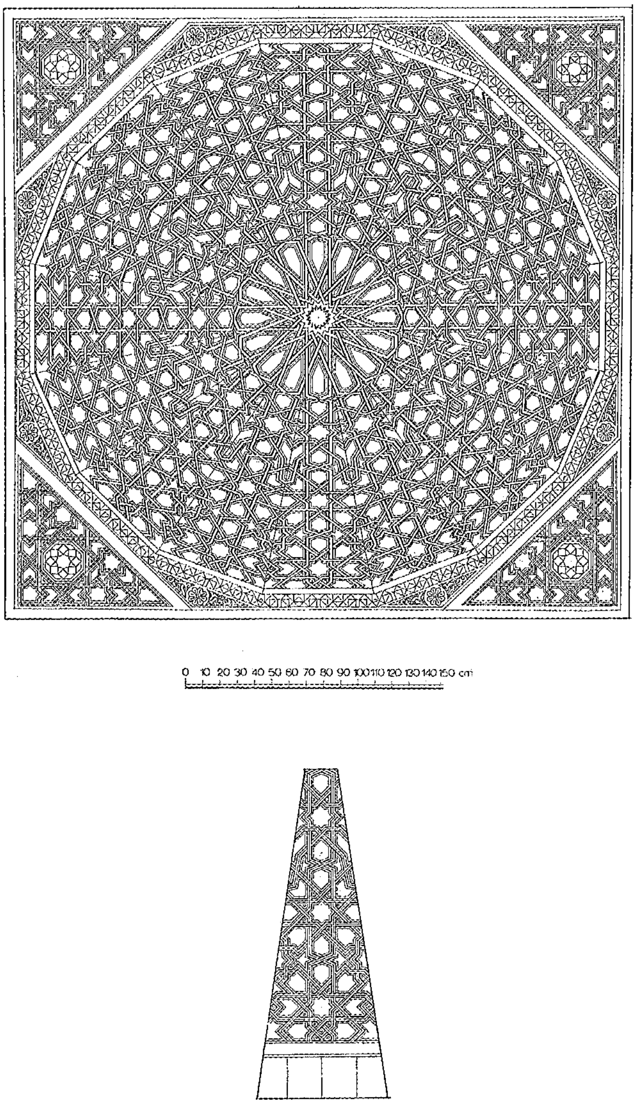


1. Three characteristic geometric ornaments from the wooden cupola of the Alhambra, 14th century.

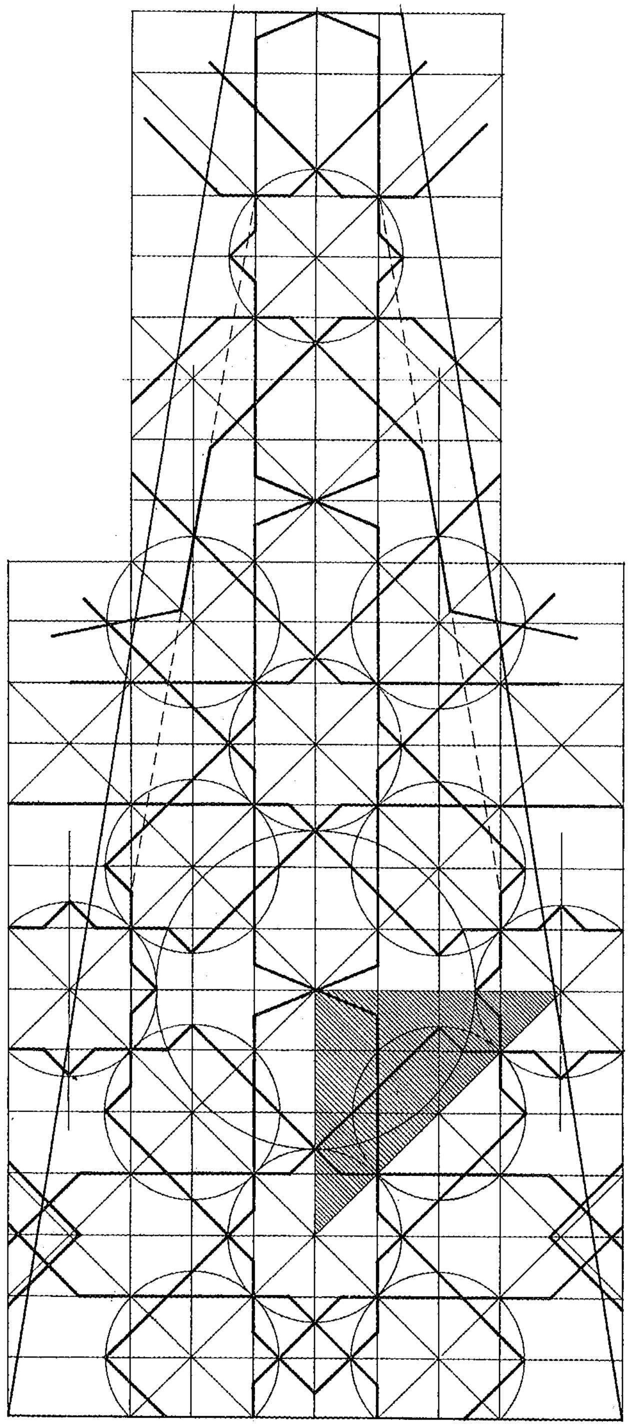
Geometric ornament, which is widespread in Islamic art, was not drawn free hand. It was designed with the help of a number of geometric methods which were not always exact but which allowed the generation of a series of variants. The most popular type of ornament, the *girih* (Persian for "knot") is represented by a design with many symmetrical axes consisting of star-shaped knots surrounded by geometric figures of various kinds. Theoretically a *girih* can be extended infinitely in all directions by repeating its smallest element or module in various rotations.

The ornament (fig. 2) on the interior apex of the cupola is based on the *girih*. Geometrical analysis reveals its invisible foundation, which proves to be a grid of square cells in which eight-pointed stars ("knots") were inserted. Each star seems to consist of two squares, turned at 45-degree angles to each other. The size of these squares equals the size of the square cells of the grid. Almost all the figures of the design in the intervals are these same stars, only this time truncated since they have lost part of their rays.

The master craftsmen did not regard the rectangle as an acceptable form for surface ornamentation, but preferred a long, narrow trapezoid. To accommodate it they transformed the design of the *girih* by giving it a beginning and an end, which in principle an ideal *girih* would not have. The essential regularity of the design was preserved, but under the lowest star a wide base was extended horizontally, and at the top elements were transformed to create the impression of a dynamic upward movement. In this way, the design, far from being neutral in relation to the architectural form it covers and decorates, facilitates the expression of its plastic and dynamic qualities or architectonics. This deliberate complication of the *girih* structure suggests



2. The ornament on the interior apex of the Alhambra cupola.



3. Drawing showing structure of the *girih*.

that the triangular module that the analysis revealed only functions in the lower central part of the design, where the star-shaped knots form the corners of a large, diagonally placed square. The rest of the design variations are quite regular, as can be seen in figure 3.

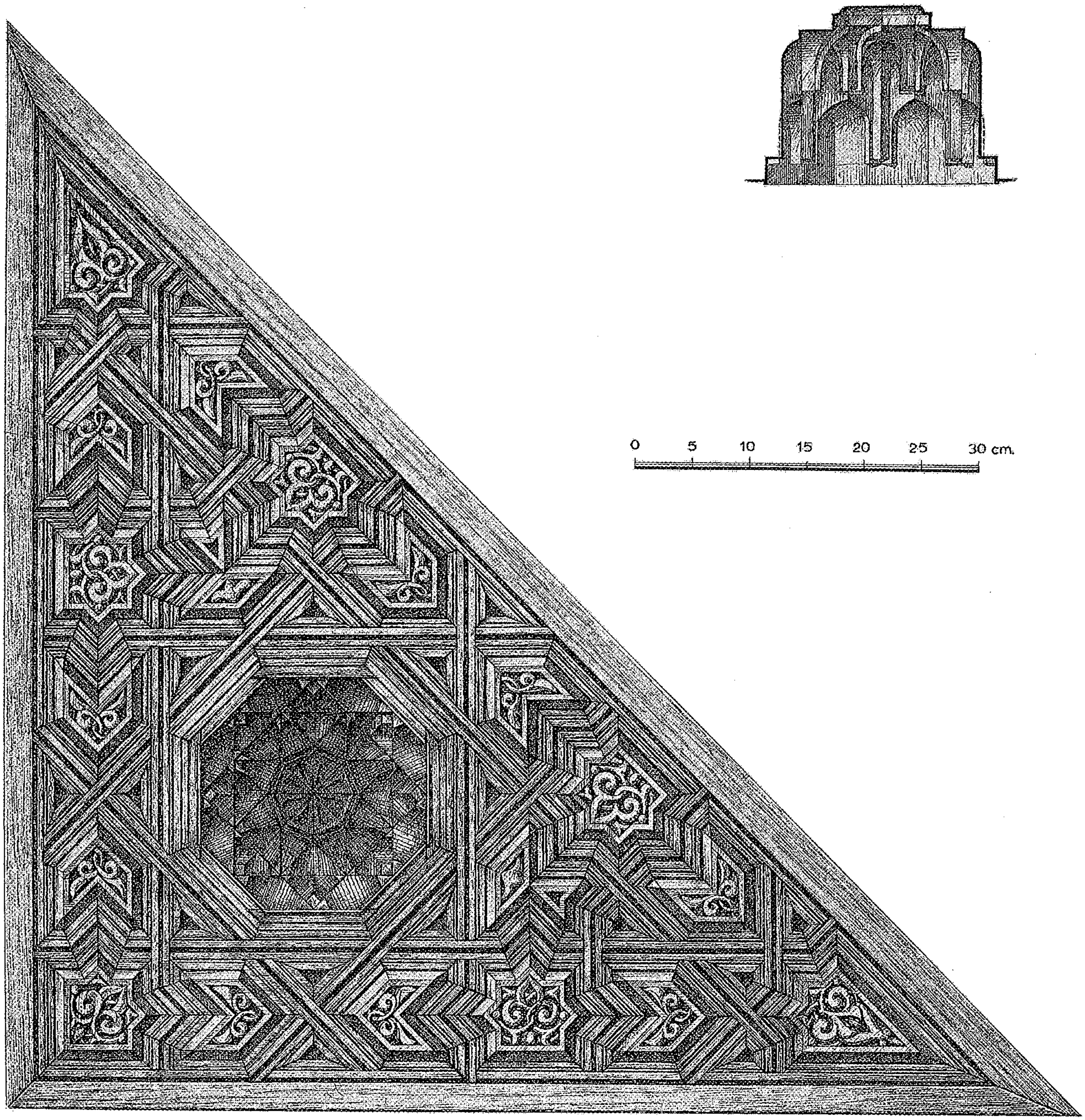
The rhythmic "orchestration" of the *girih* is reminiscent of the displacements of regular geometric systems introduced into art by "Op" artist Vasarelli six hundred years later. The most basic design, *girihs* with octagonal stars serving as knot centers, has been widely used in Islamic art for a long time. It was an international motif, the regional roots of which (like the roots of the whole art of geometrical ornamentation) are impossible to determine. One can only discuss the type, since apparently no exact model of our *girih* exists; at least I have not been able to find one. Even in the Alhambra itself, with its infinte wealth of ornamental decoration, although quite close parallels to this design can be found—for example in the Hall of the Ambassadors—the parallels are not exact. This exclusiveness is not typical, however; many *girih*s were very popular and are duplicated with great exactness in places thousands of kilometers apart. The unusualness of this one can be explained by the fact that the master craftsmen subordinated the changing structure of the ornament to the form of the surface being decorated.

The design decorating the corner triangles of the cupola's square base is of a different character. This is also in essence a *girih*, whose center forms a large eight-pointed star, in which a small eight-cornered cupola, the *ḥauzak*, is inserted. The points of the star cross over into the small stars, which are interrupted by the borders of the triangular ornamented field. Together with the geometric elements lying in between, they form the ornamental frame of the central octagonal cupola. This frame is continued down the sides of the triangle to its acute angles and then ends with small truncated stars.

In spite of its complexity, the design in figure 4 gives the impression of organized harmony, first of all because it is constructed on a system of axes. The principle axes divide the three corners of the decorative plane in half; their intersection forms the center of the main octagon. This design can only conditionally be called a *girih*; it is subordinated to the triangular form, which it is intended to decorate, and cannot go beyond those limits without altering its structure. Thus in this case the subordination to the general form of the field has a much more organic character than in the previous case, where the master craftsman transformed the geometric skeleton of the ornament more or less at will. Here there are no such transformations, and elements which do not stem organically from the basic ornamental plan are minimally attested (for example, the small stars could be squares, as is shown in the analytical drawing).

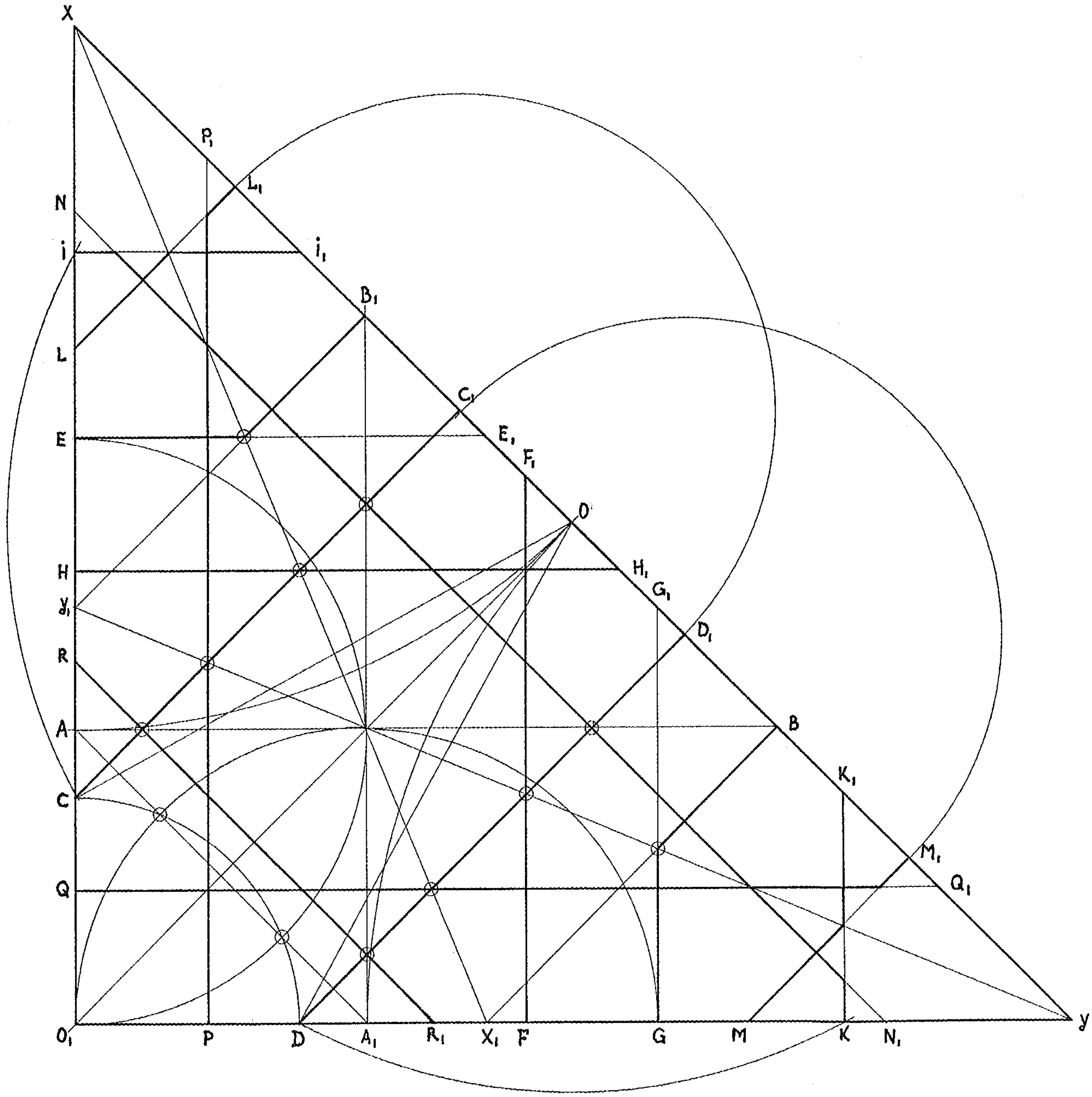
It would be comparatively easy to construct this design, gradually joining one of its elements to another until all of them lie on the corresponding axes. However the master craftsman making this decoration would not have had this possibility. He would have had to insert the ornament into the size of the triangle given to him. In other words, he had to have a method for constructing the geometric basis of the design, to have some kind of plan which would allow him to go from the general to the specific. According to that plan, the size of the triangle itself would have to determine the size and placement of the design's basic components. Geometric analysis revealed what the method for doing this was. The nature of this ornament's construction or, more exactly, its geometric framework, can be seen in figure 4.

In figure 5, the isosceles, right-angled triangle O_1XY is divided in half by the bisector 00_1. Two other angles are also divided in half by the bisectors XX_1 and YY_1. The point of intersection of the three bisectors determines the center of the design. From points X and Y we shift the dimensions AX and A_1Y_1 on the sides, which are equal halves of the hypotenuse. Drawing the lines AB and A_1B_1 parallel to the sides, we obtain the main axes of the large central star. We join the points A and A_1 with a straight line parallel to the hypotenuse. From the points A and A_1 we draw the curves having the radius AO_1 and A_1O_1 further to their intersection with AA_1 then further to the intersection with the sides at the points E and G. Through two points lying on the intersection of AA_1 with both curves, we draw a curve from O to its intersection with the sides at the points C and D. The lines CC_1 and DD_1, drawn from these points parallel to the small bisector, define two sides of the central star and the octagon lying within it. The lines EE_1 and GG_1, drawn parallel to the sides, define two angles and partially the sides of the square framing of the center. The intersection of the axes A_1B_1 with CC_1 and the axes AB with DD_1 define two points, through which the line NN_1 extends parallel to the hypotenuse. The lines PP_1 and QQ_1, parallel to the sides, are drawn through the points that result from the intersection of the bisectors XX_1 and YY_1 with CC_1 and DD_1, as well as through the points which are at the intersection of these bisectors with the already located line NN_1. These three lines—NN_1, PP_1 and QQ_1, outline the ribbons of ornament going around the inner sides of the triangle, and define three more sides of the central star. The lines HH_1 and FF_1 in defining still two more sides of the star are drawn through the intersection points of the bisectors XX_1 and YY_1 with CC_1 and DD_1. Finally, the last line forming the central star, RR_1, is defined through the points located at the intersections of CC_1 and DD_1 with the axes AB and



4. Design decorating the corner triangles of the cupola's square base.

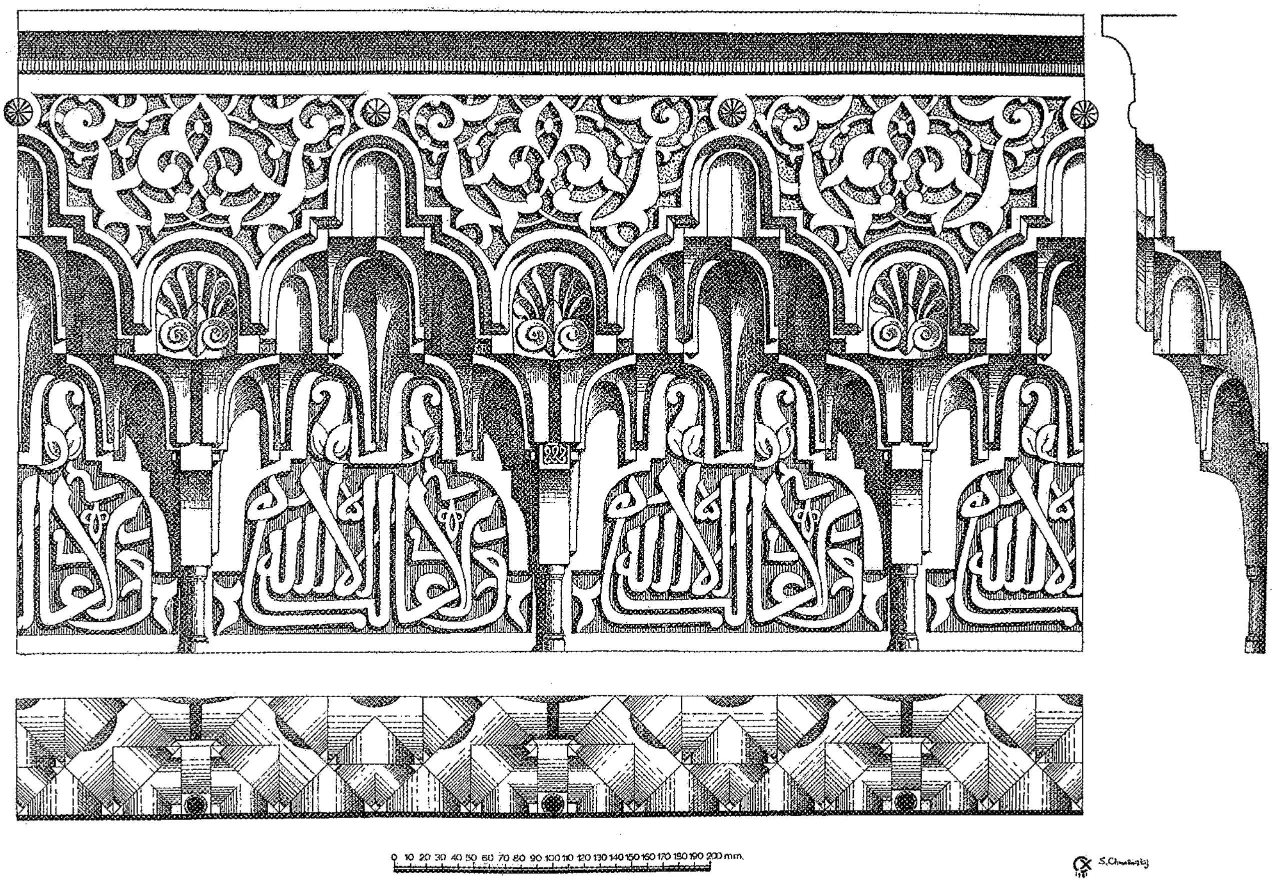
A_1B_1. The lines B_1Y_1 and BX_1 running symmetrically to EE_1 and GG_1, are drawn through the points of intersection of the latter with the bisectors XX_1 and YY_1. It is possible to draw them by joining the already mentioned points B_1 and Y_1, as well as B and X_1. The short lines II_1, LL_1, KK_1 and MM_1 remain, which form (together with the lines that run parallel to the sides of the triangle) angular truncated stars. Their position is determined by the segments C_1L_1 and D_1M_1, which are shifted from the points C_1D_1 on the



5. Geometric analysis of decoration in the corner traingle.

hypotenuse and are equal to the known segment C_1D_1, that is, to a side of the central square. The location of the points I and K on the sides is determined by the curves which are drawn from point O with a radius equal to OC (OD).

Analyzing the geometric plan's construction is undoubtedly a much easier task than creating, inventing, and composing it must have been. Nowadays it is difficult to imagine the specific geometric thought processes of the old Islamic master craftsmen which



6. Drawing representing the projection of the stalactite-like cornice structure on a horizontal plane.

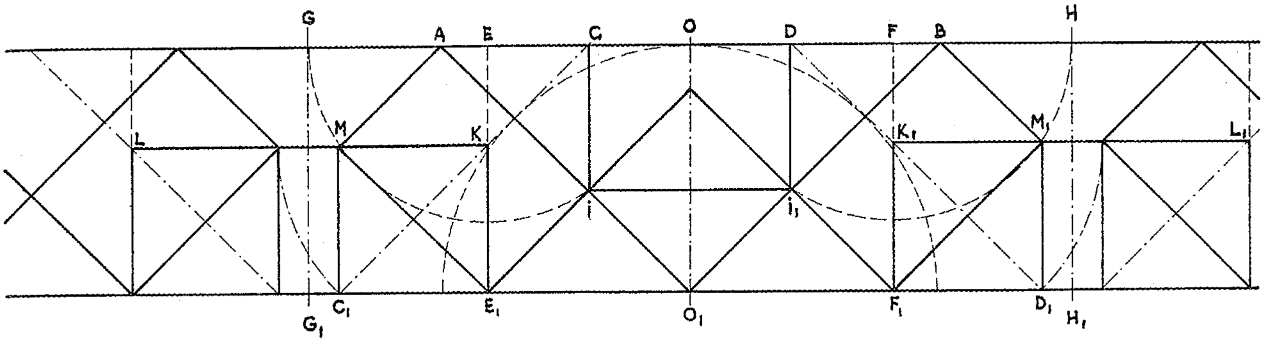
allowed them to construct, first in their imagination, then in reality, similar ornamental puzzles with their varied and complex regularities.

One such regularity in figure 4 is that the length of the right-angled elements within the frame—L_1, B_1, BM_1, GK, and EI—is equal to the diagonals of the adjacent squares, in which small stars are inserted. That also attests to the particularly profound knowledge of applied geometry in the fourteenth century which is difficult to imagine at the end of the twentieth.

All these observations also apply to the geometric system revealed by measuring the cornice of the cupola, which represents the projection of a stalactite-like cornice structure on a horizontal plane. Each part of the cornice decorating one of the sixteen edges of the cupola consists of three stalactite modules with a complicated form which rest on miniature columns and, when joined together, form two full arcs of scalloped forms covered with stalactites and two half-arcs at the corners. The complicated structure of this plastic form can be seen in figure 5.

Miniature arches and arcs, each suspended over the next in stages, are turned into a combination of geometric figures, rectangles, and triangles in the horizontal projection that together forms its own geometric design (figure 6). In this case, the design had a basic significance: it served as the pattern by which the master craftsman carved the details of the cornice out of wood and the installation plan that allowed them to join the details correctly into a single plastic whole. In creating or copying this complex structure, the master craftsman used strictly determined methods, which assured a harmonious interconnection and indivisibility of all its elements.

As in the previous case, the master craftsman estab-



7. Geometry of the stalactite cornice structure.

lished the basic measurement. Here, it is the width set for the cornice, that is, the distance the cornice projected from the wall, that served that function. We begin the reconstruction (fig. 7) by drawing the transverse axis OO_1, which corresponds on the façade to the vertical axis of the stalactite arch and lies in the center between two small columns. We assume that point O lies on the wall, that is on the interior surface of the cornice, and point O on the furthest protruding part of the upper surface. From O_1, we draw two lines at 45° angles to the intersection, with the upper lines at points A and B. From the center O_1 we draw a curve in a half-circle with the radius O_1O to the intersection with the lower wall line, then draw two tangents to this curve at a 45° angle—CC_1 and DD_1. From the points C and D we drop the perpendiculars CI and DI_1 to the intersections with O_1A and O_1B, which results in the points I and I_1 joined with a straight line. From the points I and I_1 we draw two lines at a 45° angle: one upwards to the junction with the axis OO_1, and the other downwards to the intersection with the lower edge at the points E_1 and F_1. From these points we form the perpendiculars EE_1 and FF_1, the segments of which E_1K and F_1K_1 (up to the intersection with CC_1 and DD_1) define the corresponding elements of the projection of the stalactites. These segments are equal to CI and DI_1.

When the curves with the radius EI and FI_1 are drawn from the centers E and F, we find the points G and H at the upper edge. The perpendiculars GG_1 and HH_1 that are dropped from them form the axes of the small columns and the elements which rest on them. These axes border on the continually repeating parts of the cornice. The space from these axes to the perpendiculars C_1M and D_1M_1 formed from the points C_1D_1 defines half the width of the modules, which rest on the small columns. The segments MK and M_1K_1, which are drawn from the points K and K_1, define the length of these modules in the basic plan (their extension) and form, together with the diagonals E_1M and F_1M_1, the stalactite elements of the cornice, which are inserted into the square in the plan. When we join the already known points, we get the segments AM and BM_1, which close the structure of the rectangles AIE_1M and $BI_1F_1M_1$ and complete the construction of the repetitive portion of the cornice.

The stalactite cornice, which looks so complex in the façade, in the plan is reduced to a combination of simple geometric figures: squares which are diagonally divided in two, and rectangles into which halves of the squares are set. All the squares are equal in size, and in the rectangles one side is equal to the side of the square, and the other is equal to its diagonal. The interrelation of proportions ensures a harmonious proportionality of all stalactite structures of the cornice, whereby the figures in the plan which have the forms of triangles and rectangles with a corner cut out of one side in reality depict small arches of stalactite cells.

With the help of analogical methods, all the stalactite systems were created not only for cornices, but for covering niches, cupolas and arches, which were popular in Islamic architecture. The principle of the geometric basis of stalactite structures (at times unbelievably complicated) is the same everywhere it was used—in Spain, Egypt, Iran, and India. Whatever its degree of complexity it yields to geometric analysis. However, the art of the creation, or "composition," of such structures presupposing a unique "geometric imagination" has probably been lost forever.

West Berlin
(translated from the Russian)