Foundations, Gift 10

Due Monday, November 18

This homework must be done individually. Remember to follow Math department's guidelines for collaboration on homework. Please write your solutions independently and neatly. Typesetting in LaTeXis highly appreciated and encouraged.

1. Let $x, y, z \in \mathbb{Z}$ and $z \neq 0$. Determine whether the following statements are true or false. Provide a proof or a counterexample.

   (a) If $z \mid (x + y)$ then $z \mid x$ and $z \mid y$

   (b) If $z \mid (x + y)$ then $z \mid x$ or $z \mid y$

   (c) If $z \mid (x + y)$ then either z divides both x and y, or z divides neither x nor y

   (d) If $z \mid (x \cdot y)$ then $z \mid x$ or $z \mid y$

   (e) If $z \mid x$ or $z \mid y$ then $z \mid (x \cdot y)$

   (f) If $z^2 \mid x^2$ then $z \mid x$. (Hint: use the Fund Thm of Arithmetic, Thm 6.5.3)

2. Let $x, y, z \in \mathbb{Z}$ such that $x^2 + y^2 = z^2$. Show that if z is even, then both x and y are even.

3. Given any 3-digit number (e.g., 207), form the 6-digit number by repeating the digits of the original number twice (i.e., 207207). Prove that the resulting number is always composite. In fact, show that it always has at least 3 distinct prime divisors.