

Foundations, Gift 1

Due Monday, Sep 7

This homework must be done individually. You are not allowed to use any sources other than the textbook, class notes, your professor, and the tutor for this course. Remember to follow Math department's guidelines for collaboration on homework, and the AI policy for the course. Please write your solutions neatly. Typesetting in L^AT_EX is highly appreciated and encouraged.

1. Do problem 2 on page 37 of the textbook
2. Do problem 3 on page 37 of the textbook with the following additional parts. For each statement write out its converse, its contrapositive, and its negation and determine their truth values.
3. Do problem 4 on page 37 of the textbook
4. Do problem 8 on page 38 of the textbook, and give a specific example.
5. Do problem 9 on page 38 of the textbook.
6. Negating statements involving quantifiers. Given the following fact

Theorem 1. *Let $P(x)$ be a predicate in some universe of discourse. Then*

(a) $\sim (\forall x)P(x)$ is logically equivalent to $(\exists x)(\sim P(x))$

(b) $\sim (\exists x)P(x)$ is logically equivalent to $(\forall x)(\sim P(x))$

write down negations of the following statements. Your negated statements should not contain the symbol $\sim$.

(a) $(\forall x)[x > 0 \Rightarrow (\exists y)(y < 0 \wedge xy > 0)]$

(b) For any $\epsilon > 0$, there exists $\delta > 0$ such that for all $a, b \in S$, if $|a - b| < \delta$ then $|f(a) - f(b)| < \epsilon$. (Here f is a real-valued function whose domain contains S .)