Problem 2: Orthogonal Matrices

A square matrix M over a field F is called λ-orthogonal if there exists a $\lambda \in F$ such that $MM^t = \lambda I$ where M^t is the transpose of M and I is the identity matrix. Does there exist a 3×3, 2-orthogonal matrix over $\mathbb{Z}_5$? Determine all non-zero $\lambda \in \mathbb{Z}_5$ such that a 3×3, λ-orthogonal matrix over $\mathbb{Z}_5$ exists that is not a scalar multiple of the identity matrix.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 1/18/2026. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Friday, January 30, 2026. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.