

Problem 1: New Year's Sum

Let ζ be a primitive 2026^{th} root of unity in $\mathbb{C}$. Define $S = \sum_{k=1}^{2025} \frac{1}{1 - \zeta^k}$

- a) Show that $S \in \mathbb{R}$
- b) Compute the exact value of S

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 1/10/2026. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Sunday, January 18, 2026. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.