Problem 6: Power of Counting

Give a combinatorial argument (such as counting something in more than one way or using the bijection principle; a purely algebraic proof will not be accepted) for each of the following identities:

1. $\binom{n}{s} = \frac{n}{s}\binom{n-1}{s-1}$
2. $\sum_{i=0}^{n} \binom{n}{i}\binom{n}{n-i} = \binom{2n}{n} = \sum_{i=0}^{n} \binom{n}{i}^2$

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 3/29/2024. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by 5 pm on Friday, April 12, 2024. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.