Problem 4: Sum Divisible by n?

Let a_1, a_2, \ldots, a_n be a set of n (not necessarily distinct) integers. Is it true that there always exists a subset of these integers with sum divisible by n?

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 2/16/2024. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Friday, March 1, 2024. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.