Problem 1: Polynomial with Integer Coefficients

Does there exist a polynomial $f(x)$ with integer coefficients such that $f(3) = 2024$ and $f(7) = 2027$?

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.