## Problem 6: Migrating Birds

Power $(P)$ is the rate at which energy $(E)$ is consumed per unit time. Ornithologists have found that the power consumed by a certain pigeon flying at velocity $v \mathrm{~m} / \mathrm{s}$ is described well by the function $P(v)=17 v^{-1}+10^{-3} v^{3} \quad J / s$. Assume that the pigeon can store $5 \times 10^{4} \quad J$ of usable energy as body fat.

1. Find the velocity that minimizes the power consumption.
2. Show that a pigeon flying at velocity $v$ and using all of its stored energy can fly a total distance $D(v)=\left(5 \times 10^{4}\right) v / P(v)$.
3. Migrating birds are smart enough to fly at the velocity that maximizes distance traveled rather than minimizes power consumption. Show that the velocity $v_{d m a x}$ that maximizes the total distance satisfies $P^{\prime}(v)=P(v) / v$.
4. Find $v_{d m a x}$ and the maximum total distance that the bird can fly.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 3/20/2021. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on April 2, 2021. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.

