Problem 5: Hexagony

Consider a hexagon with the area of 6 sq-ft. Choosing a subset of the vertices of this hexagon yields a polygon formed by connecting those vertices so that no two edges cross.

Is it possible to find six such polygons whose areas are 1,2,3,4,5, and 6 sq-ft respectively? If so, what is the smallest total number of edges needed? For example, if you can find a triangle for each of these 6 areas, then the total number of edges is $6 \cdot 3 = 18$.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 3/14/2020. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by noon on 3/22/2020.