Problem of the Week-5

Given that

$$x + x^{-1} = 2\cos(\frac{\pi}{5}) = \frac{1 + \sqrt{5}}{2}$$

find the value of

$$x^{2000} + x^{-2000}$$

As always, show your work, fully explain and justify your answer. No calculators or computers.

Hint: Consider complex nth roots of unity.