Problem 7: A System of Equations

Consider the real numbers x, y that satisfy the equations

$$x + y = x^3 + y^3 = x^5 + y^5$$

What are the possible values of $x + y$? Fully explain your answer and show your steps.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted. The use of generative AI tools is not allowed.