Problem 5: A Fun Fact About Binomial Coefficients

Let n be a positive integer and consider the set of binomial coefficients

$$\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}.$$

Show that for each positive integer n, the number of odd binomial coefficients is a power of 2. Fully explain your answer and show your steps.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted. The use of generative AI tools is not allowed.

Posting Date 10/24/2025. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Fri, Nov 7, 2025. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.