Problem 3: Fractions

Let p, q, r be positive integers that are all mutually coprime. Suppose P, Q, R are also integers such that $\dfrac{P}{p} + \dfrac{Q}{q} + \dfrac{R}{r}$ is an integer. Show that each of the fractions $\dfrac{P}{p}, \dfrac{Q}{q}$, and $\dfrac{R}{r}$ is an integer. Fully explain your answer and show your steps.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted. The use of generative AI tools is not allowed.

Posting Date 9/26/2025. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Fri, Oct 10, 2025. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.