Problem 4: An Increasing Function

Let f be a real-valued function that is continuous and differentiable on $[0, \infty)$ whose derivative is increasing on $[0, \infty)$ with $f(0) = 0$. Define

$$g(x) = \begin{cases} \frac{f(x)}{x}, & x > 0 \\ f'(0), & x = 0 \end{cases}$$

Show that g is an increasing function.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 10/7/2021. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by 4 pm on Friday, October 22, 2021. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.