Problem 1: Find $f(2021)$

A function f is defined for all positive integers and satisfies

$$\begin{cases} f(1) = 2021, \\ f(1) + f(2) + \cdots + f(n) = n^2 f(n) \quad \text{for all } n > 1. \end{cases}$$

Calculate the exact value of $f(2021)$.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 8/28/2021. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by 4 pm on Friday, Sep 10, 2021. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.