Problem 2: Coloring a Polygonal Rolling-pin

A polygonal rolling-pin n-die has n-2 long rectangular faces and two regular (n-2)-gons to provide a cap at each end of the rolling-pin. A proper coloring of the n-die requires that each face has a color that is different from all of its adjacent faces, edges and vertices, and further that each edge has a color different from all of its adjacent edges and vertices. For each n > 4, find

Min(n): the minimum number of colors needed to color the *n*-die properly.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 9/8/2019. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 5 pm on 9/20/2019.