Problem 2: A Combinatorial Identity

Find a closed formula for the sum

$$\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + \dots + \binom{n}{k}$$

for all non-negative integers k and n, that is, write it in the form $\binom{r}{s}$, and give a combinatorial argument (an argument based on counting) to justify your answer. A purely algebraic explanation/justification will not be accepted.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 9/14/2018. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 9/27/18.