

Problem 2: Triangular Numbers

Triangular numbers are so named because they can be arranged in a triangular configuration. The first few triangular numbers are 1,3,6,10,15,21,28,36,...

Consider the following pattern

$$1 = 1$$

$$1 + 5 = 6$$

$$1 + 7 + 7 = 15$$

$$1 + 9 + 9 + 9 = 28$$

$$1 + 11 + 11 + 11 + 11 = 45$$

⋮

This is a pattern that gives every other triangular number (the first, third, fifth, ...) Find a closed formula for the n^{th} term of this sequence. Then find a similar pattern for even indexed triangular numbers (second, fourth, sixth, ...) and find a closed formula for the n^{th} term of that sequence.

As always, show your work, fully explain and justify your answer.