#1. Let \( N = \# \) of floors at which the elevator makes a stop to let out one or more people. 

\( N \) is a counting variable which can be written as the sum of ten indicator variables (just like binomial R.V.'s)

\[
N = \sum_{i=1}^{10} I \left( \text{at least one person chooses floor } i \right)
\]

\[
E[N] = E \left[ \sum_{i=1}^{10} I(F_i) \right], \text{ where } F_i = \{ \text{at least one person chooses floor } i \}
\]

\[
= \sum_{i=1}^{10} E[I(F_i)]
\]

\[
= \sum_{i=1}^{10} P(\text{at least one person chooses floor } i)
\]

Now, for each \( i \)

\[
P(F_i) = 1 - P(\text{ nobody chooses floor } i) = 1 - \left( \frac{9}{10} \right)^{10} \approx 0.7176
\]

\[
E[N] = \sum_{i=1}^{10} \left[ 1 - \left( \frac{9}{10} \right)^{12} \right] = 10 \left[ 1 - \left( \frac{9}{10} \right)^{12} \right] = 7.176
\]

#2. a. Number of cookies in each bowl

<table>
<thead>
<tr>
<th>Choc. Chip</th>
<th>Bowl 1</th>
<th>Bowl 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>20</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plain</th>
<th>Bowl 1</th>
<th>Bowl 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>20</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>40</td>
<td>80</td>
<td></td>
</tr>
</tbody>
</table>

Relative Frequency of Cookies in each bowl

<table>
<thead>
<tr>
<th>Choc. Chip</th>
<th>Bowl 1</th>
<th>Bowl 2</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.0833</td>
<td>0.475</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plain</th>
<th>Bowl 1</th>
<th>Bowl 2</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.375</td>
<td>0.250</td>
<td>0.625</td>
<td></td>
</tr>
</tbody>
</table>

\[
P(B1 | \text{ Plain}) = \frac{P(\text{ Plain } | B1) P(B1)}{P(\text{ Plain } | B1) P(B1) + P(\text{ Plain } | B2) P(B2)}
\]

\[
= \frac{0.75 (0.5)}{0.75 (0.5) + 0.5 (0.5)} = \frac{0.375}{0.625} = 0.60
\]
#2 (b) \[ P(BC | pos) = \frac{P(pos | BC) P(BC)}{P(pos | BC) P(BC) + P(pos | BC^c) P(BC^c)} \]
\[= \frac{(0.8)(0.01)}{(0.8)(0.01) + (0.096)(0.99)} \]
\[= \frac{0.008}{0.008 + 0.095} = \frac{0.008}{0.103} = 0.0777 \]
\[\text{or } 7.77\%\]

#3. Let \( H = \{ \text{# of hearts in a 5 card hand} \} \)
\[ P(H=2) = \frac{\binom{13}{2} \left( \frac{\binom{11}{3}}{\binom{52}{5}} \right)}{\binom{52}{5}} = \frac{78 \times 9,139}{2,598,960} \]
\[= 0.27428 \]

#4. Let \( X = \# \text{ of male geckos hatched}. \)
\( X \sim B(20, p = .35) \)
(a) \( E[X] = np = 20(.35) = 7 \)
(b) \( P(X \geq 5) = 1 - P(X < 5) = 1 - P(X \leq 4) = 1 - 0.1182 = 0.8818 \)
(c) \( P(X=7) = \binom{20}{7}(.35)^7(1-.35)^{20-7} = 0.1844 \)
(d) Use geometric dist or \( P(F_1F_2F_3F_4M_5) = .65^4(.35) = .0625 \)
(e) If \( n = 2000 \), then \( npq = 2000(.35)(.65) = 455 > 5 \). Thus, the normal approximation would be appropriate. The Poisson approximation is useful when \( p \) is close to 1 (or close to 0). No approximations should not be used in this situation.
#5. a. Use multinomial dist. with 6 categories and $p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = \frac{1}{6}$.

$P(6\text{ faces in 6 rolls}) = \frac{6!}{6!6!1!1!1!1!1!1!1!} \left( \frac{1}{6} \right)^6 = \frac{5}{324} = .0154$

b. $P(41's\text{ and }22's) = \frac{6!}{4!2!1!1!1!1!} \left( \frac{1}{6} \right)^4 \left( \frac{1}{6} \right)^2 = \frac{5}{1552} = .003$

#6. a. As we have seen with previous dice problems, the distribution of the sum is symmetric. The center of the distribution is the most likely value. Thus, the most probable sum is $E[I_1+I_2] = \frac{n+1}{2} + \frac{n+1}{2} = n+1$.

Alternatively, we could consider the sample space:

![Sample Space Diagram](image)

- $P(\Sigma_1+\Sigma_2 = 2) = P(\Sigma_1+\Sigma_2 = 2) = \frac{1}{36}$
- $P(\Sigma_1+\Sigma_2 = 3) = P(\Sigma_1+\Sigma_2 = 3) = \frac{2}{36}$
- $P(\Sigma_1+\Sigma_2 = 4) = P(\Sigma_1+\Sigma_2 = 4) = \frac{3}{36}$
- $P(\Sigma_1+\Sigma_2 = n+1) = \frac{n}{36}$

Most probable value $= (n+1)$

b. $X\{\begin{array}{ccccccc}
1 & 2 & 3 & \cdots & n \\
\text{prob} & \frac{1}{n} & \frac{1}{n} & \frac{1}{n} & \cdots & \frac{1}{n} \\
\end{array}\}

f_x(x) = \begin{cases} \frac{1}{n}, & x = 1, 2, \ldots, n \\ 0, & \text{otherwise} \end{cases}$
# 6 c. Consider the case where \( n = 1 \):

\[
E[X^2] = \frac{1}{1} (1^2) = 1 = \frac{(1+1)(2(1)+1)}{6} = \frac{2*2}{6} \quad \checkmark
\]

**Induction hypothesis:**

\[
E[X^2] = \frac{(k+1)(2k+1)}{6}
\]

Prove the relationship holds for \( n = k+1 \):

\[
E[X^2] = \frac{1^2}{k+1} + \frac{2^2}{k+1} + \cdots + \frac{k^2}{k+1} + \frac{(k+1)^2}{k+1}
\]

\[
= \frac{1}{k+1} \left( 1^2 + 2^2 + \cdots + k^2 + (k+1)^2 \right)
\]

\[
= \frac{k}{k+1} \left( \frac{1^2 + 2^2 + \cdots + k^2}{k} + \frac{(k+1)^2}{k} \right) + k+1
\]

**Induction hypothesis**

\[
= \frac{k}{k+1} \left( \frac{(k+1)(2k+1)}{6} \right) + k+1
\]

\[
= \frac{k(2k+1) + 6(k+1)}{6}
\]

\[
= \frac{2k^2 + k + 6k + 6}{6}
\]

\[
= \frac{2k^2 + 7k + 6}{6}
\]

\[
= \frac{(k+2)(2k+3)}{6}
\]

\[
= \frac{(k+1+1)(2(k+1)+1)}{6} \quad \checkmark
\]