Probability (Math 336) - Hartlaub
Exam 1 - October 6, 2000

To receive maximum credit you must show your work. GOOD LUCK!

1. A bag of 50 jelly beans includes exactly eight that are green. Suppose we select beans one at a time until we get one that is green. Find the probability that we select exactly four beans. (15)

2. License plates are issued at random, each having three letters followed by three digits. The initial distribution of plates includes all those on which the first letter is either A or B. What is the probability that you get a plate on which the three letters are the same and the three digits are the same? (15)

3. A study of Georgia residents suggests that those who worked in shipyards during World War II were subjected to a significantly higher risk of lung cancer. It was found that approximately 22% of those persons who had lung cancer worked at some prior time in a shipyard. In contrast, only 14% of those who had no lung cancer worked at some previous time in a shipyard. Suppose that the proportion of all Georgians living during World War II who have or will have contracted lung cancer is .04%. Find the percentage of Georgians living during the same period who will contract (or have contracted) lung cancer, given that they have at some prior time worked in a shipyard. (20)

4. An auditor samples 100 of a firm’s travel vouchers to ascertain what percentage of the whole set of vouchers are improperly documented. What is the approximate probability that more than 30% of the sampled vouchers are improperly documented, if in fact, only 20% of all of the vouchers are improperly documented? If you were the auditor and observed more than 30% with improper documentation, what would you conclude about the firm’s claim that only 20% suffered from improper documentation? Why? (20)

5. If P(A) > 0, P(B) > 0, and P(A) < P(A|B), show that P(B) < P(B|A). (20)

6. If A₁, A₂, and A₃ are three events and P(A₁A₂) = P(A₁A₃) ≠ 0, but P(A₂A₃) = 0, show that P(at least one Aᵢ) = P(A₁) + P(A₂) + P(A₃) − 2P(A₁A₂). (20)

7. Use mathematical induction to prove the multiplication rule for n mutually independent events, \[ P\left(\bigcap_{i=1}^{n} E_i\right) = \prod_{i=1}^{n} P\left(E_i\right). \] (20)

Extra Credit:

Use an element proof to show that \( A \cup B = (A \cap B^c) \cup (A \cap B) \cup (A^c \cap B) \). (15)