1. In the blue and fin whale problem we discussed previously, we used a logistic model of population growth, where the growth of population \( P \) in the absence of interspecies competition is

\[
g(P) = rP \left( 1 - \frac{P}{K} \right).
\]

In this problem, we will be using a more complex model,

\[
g(P) = rP \left( \frac{P - c}{P + c} \right) \left( 1 - \frac{P}{K} \right),
\]

in which the parameter \( c \) represents a minimum viable population level below which the growth rate is negative. Assume that \( \alpha = 10^{-8} \) and that the minimum viable population level is 3,000 for blue whales and 15,000 for fin whales.

(a) Can the two species of whales coexist?

(b) Sketch the vector field and classify each equilibrium point as stable or unstable.

(c) Assuming that there are currently 5,000 blue whales and 70,000 fin whales, what does this model predict about the future of the two populations?

(d) Suppose that we have underestimated the minimum viable population for the blue whale, and that it is actually closer to 10,000. Now what happens to the two species?

(e) Investigate the sensitivity of your conclusions to the estimated value of one of the parameters. Explain your choice of parameter.

\[1\] from *Mathematical Modeling*, by Mark M. Meerschaert and from *Concepts of Mathematical Modeling*, by Walter J. Meyer.
2. (a) Show that the sensitivity of \( r \) to \( \varphi \) in Aristarchus' model is approximately 29.9726.

(b) Poseidonius estimated the circumference of the earth in the following way. He observed that on a certain day the star Canopus appeared to just graze the horizon when viewed from Rhodes, whereas at Alexandria it appeared \( 7 \frac{1}{2}^{\circ} \) above the horizon. Taking the distance between Rhodes and Alexandria as 3750 stadia and assuming lines \( AC \) and \( RC \) to be essentially parallel, Poseidonius obtained 180,000 stadia for the circumference. Work out the details of his calculation, and discuss the robustness of his model.

(c) **Journal Assignment** Several excellent observations were made during our discussion in class about the robustness of the models created by Aristarchus and Eratosthenes. Write your own thoughts judging the robustness of the models.