A Quick Review of Integration

Area Definition.

\[ \int_{a}^{b} f(x) \, dx = (\text{area above x-axis}) - (\text{area below x-axis}) \]

We’ll talk more about the area definition of the definite integral later in the course.

Fundamental Theorem of Calculus, Part 2.

\[ \int_{a}^{b} f(x) \, dx = F(b) - F(a), \]

where \( F(x) \) is any anti-derivative of \( f(x) \), i.e. \( F(x) \) is any anti-derivative of \( f(x) \), i.e. any function such that \( F'(x) = f(x) \).

Indefinite Integrals (also called Anti-derivatives).

\[ \int f(x) \, dx = F(x) + C, \]

where \( F(x) \) is any anti-derivative of \( f(x) \).

Examples

1. Find the area between the graph of \( f(x) = x^2 \) and the x-axis over the interval \([0, 4]\).

2. Show that the area under one arch of the sine curve \( y = \sin x \) is 2.
3. Evaluate $\int \frac{3}{x+1} \, dx$.

4. Evaluate $\int_0^\pi (1 + \cos x) \, dx$.

5. Evaluate $\int \frac{2}{x^2} \, dx$.

6. Evaluate $\int e^{-3x} \, dx$. 
Integration by Substitution

Evaluate the following integrals.

1. \[ \int 2 \cos (2x + 5) \, dx \]

2. \[ \int \sqrt{3x - 5} \, dx \]

3. \[ \int -2x \sqrt{1 - x^2} \, dx \]
4. \( \int \tan^2 x \sec^2 x \, dx \)

5. \( \int \tan x \, dx \)

6. \( \int \frac{1}{x} \cos \frac{1}{x} \, dx \)
7. \( \int_0^2 4x^3 \sqrt{x^4 + 7} \, dx \)

8. \( \int_\pi^{2\pi} 2x \cos x^2 \, dx \)