Homework Set Hmwk1 due 4/5/09 at 6:00 AM

This set covers sections 5.1-5.5 of the text.

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

1.(1 pt) Approximate $\int_0^{\pi/2} \sin(x)dx$ by computing $L_f(P)$ and $U_f(P)$, using the partition $\{0, \pi/6, \pi/4, \pi/3, \pi/2\}$.

Your answers should be accurate to at least 4 decimal places.

$L_f(P)$ = __________

$U_f(P)$ = __________

You may include a formula as an answer.

Note that this is Problem 10 in Section 5.1 in the text.

2.(1 pt) Approximate $\int_0^{\pi/2} x\sin(x)dx$ by computing $L_f(P)$ and $U_f(P)$, using the partition $\{0, \pi/6, \pi/4, \pi/3, \pi/2\}$.

Your answers should be accurate to at least 4 decimal places.

$L_f(P)$ = __________

$U_f(P)$ = __________

You may include a formula as an answer.

Note that this is Problem 18 in Section 5.1 of the text.

3.(1 pt) Approximate the definite integral

$$\int_4^6 |5-t|\,dt$$

using midpoint Riemann sums with the following partitions:

(a) $P = \{4, 5, 6\}$. Then midpoint Riemann sum = __________

(b) Using 4 subintervals of equal length. Then midpoint Riemann sum = __________

Note that this problem is similar to problems 7 and 10 of Section 5.2 in the text, except you are asked to use midpoint sums instead of upper and lower sums.

4.(1 pt) Use the Midpoint Rule to approximate the integral

$$\int_9^{13} (6x-4x^2)dx$$

with n=3.

This is similar to Problems 29 and 31 of Section 5.2 of the text.

5.(1 pt) Let $\int_0^{4.5} f(x)dx = 9, \int_0^{1.5} f(x)dx = 7, \int_3^{4.5} f(x)dx = 10$.

Find $\int_{1.5}^{3} f(x)dx =$ __________

and $\int_3^{1.5} (9f(x)-7)dx =$ __________

This is similar to Problems 9-12 in Section 5.3 of the text.

6.(1 pt) Find the mean value of the function $f(x) = |6-x|$ on the closed interval $[4, 8]$.

mean value = __________

Note that this problem is similar to Problem 20 in Section 5.3 of the text. You will first need to find the integral of $f(x)$: find this by interpreting the integral as the sum of areas of two elementary geometric figures.

7.(1 pt) You are given the four points in the plane $A = (6,-7)$, $B = (9,6)$, $C = (12,-8)$, and $D = (15,1)$. The graph of the function $f(x)$ consists of the three line segments AB, BC and CD. Find the integral $\int_6^{15} f(x)\,dx$ by interpreting the integral in terms of sums and/or differences of areas of elementary figures.

$\int_6^{15} f(x)\,dx =$ __________

8.(1 pt) The following sum

$$\frac{1}{1+\frac{3}{n}}\frac{3}{n} + \frac{1}{1+\frac{6}{n}}\frac{3}{n} + \frac{1}{1+\frac{9}{n}}\frac{3}{n} + \cdots + \frac{1}{1+\frac{3n}{n}}\frac{3}{n}$$

is a right Riemann sum for a certain definite integral

$$\int_1^b f(x)\,dx$$

using a partition of the interval $[1, b]$ into n subintervals of equal length.

Then the upper limit of integration must be: b = __________

and the integrand must be the function $f(x)$ = __________

This is an easier (less abstract) version of Problem 33 in Section 5.3 of the text. Recognizing Riemann sums is very important in applications of integral calculus to science and engineering.

9.(1 pt) **Note:** You can get full credit for this problem by just entering the answer to the last question correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

A particle moves along a straight line in such a way so that its velocity v at position x is given by the formula

$$v = x^2.$$

Find the time it takes the particle to travel from $x = 1$ to $x = 6$.

We first find an approximate answer to our problem by subdividing the interval from $x = 1$ to $x = 6$ into n equal subintervals, each of length $\Delta x =$ __________

The first subinterval is from $x =$ __________ to $x =$ __________ . Approximate the velocity of the particle over this subinterval by its velocity at the right hand endpoint. Then the time it takes the particle to travel over the first subinterval is approximately __________ .

The second subinterval is from $x =$ __________ to $x =$ __________ . Approximate the velocity of the particle over this subinterval by its velocity at the right hand endpoint. Then the time it takes the particle to travel over the second subinterval is approximately __________ .

The third subinterval is from $x =$ __________ to $x =$ __________ . Approximate the velocity of the particle over this subinterval by its velocity at the right hand endpoint. Then the time it takes the particle to travel over the third subinterval is approximately __________ .

In general, the k-th subinterval is from $x =$__________ to $x =$________ . Approximate the velocity of the particle over this subinterval by its velocity at the right hand endpoint. Then the time it takes the particle to travel over the first subinterval is approximately ______________________________ .

Summing up these approximations over all the subintervals gives us a right Riemann for the definite integral $\int_a^b f(x)dx$, where $a =$____________ , $b =$____________ , and $f(x) =$______________________________ .

As $n \to \infty$, these approximations become more and more accurate, and in the limit give us the exact answer. We conclude that the time it takes the particle to travel from $x = 1$ to $x = 6$ is ____ .

10.(1 pt) The value of $\int_0^{-8}(x-3)^2dx$ is

____

This is similar to Problems 11-25 in Section 5.4 of the text.

11.(1 pt) Evaluate the definite integral

$$\int_1^5 \frac{9}{\sqrt{x}}dx$$

____

This is similar to Problem 6 in Section 5.5 of the text.

12.(1 pt) $\int_4^7 \frac{5x^2+2}{x^2}\,dx =$ ________________________

This is similar to Problem 32 in Section 5.5.

13.(1 pt) $\int_b^{2b} x^8\,dx =$

____

This is similar to Problem 16 in Section 5.4 of the text.

14.(1 pt) The velocity function is $v(t) = -t^2 + 6t - 8$ for a particle moving along a line. Find the displacement (net distance covered) of the particle during the time interval [-1,5].

displacement = ______________________________

This is similar to Problem 60 in Section 5.4 of the text.

15.(1 pt) Evaluate the definite integral

$$\int_2^3 \left(\frac{d}{dt}\sqrt{2+5t^4}\right) dt$$

using the Fundamental Theorem of Calculus.

You will need accuracy to at least 4 decimal places for your numerical answer to be accepted. You can also leave your answer as an algebraic expression involving square roots.

$\int_2^3 \left(\frac{d}{dt}\sqrt{2+5t^4}\right) dt$
= ____

This is similar to Problems 39 and 40 in Section 5.4 of the text.

16.(1 pt) Find the derivative of the following function

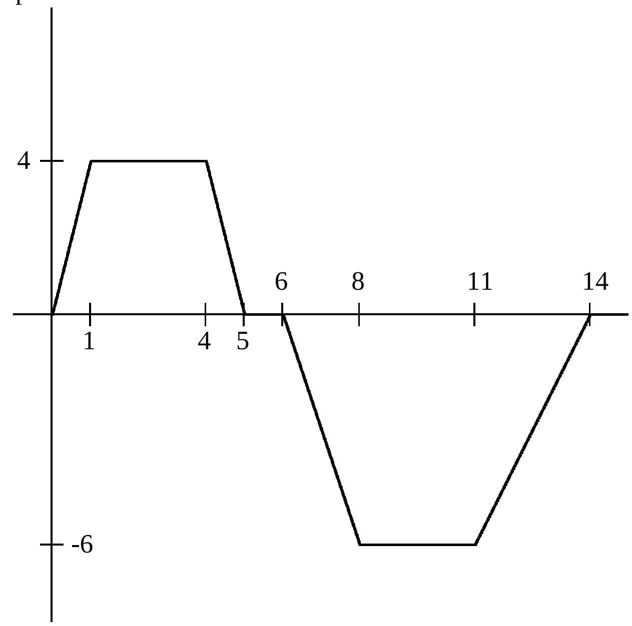
$$F(x) = \int_{x^2}^{x^4} (2t-1)^3\,dt$$

using the Fundamental Theorem of Calculus.

$F'(x) =$ ____________________________________

This is similar to Problems 3, 4, 5, 8, 9 and 10 of Section 5.4 in the text.

17.(1 pt) An airplane takes off at sea level, flies for 14 minutes in a dense fog, then crashes. Suppose the height of the airplane at time t minutes after takeoff is given by a function $h(t)$. The **derivative** of this function, $h'(t)$, is shown on the graph below.



The vertical axis of the graph is displayed in units of thousand ft/min. The horizontal axis is displayed in units of minutes. Using the information displayed in the graph, answer the following questions:

The maximum height reached by the plane is ________ feet.

At $t = 10$ minutes, the planes altitude is __________ . Your answer must be one of the following: increasing, decreasing, or holding steady. (No abbreviations.)

The plane crashes at an altitude of ____________ feet.

Which of the following objects did the plane crash into?

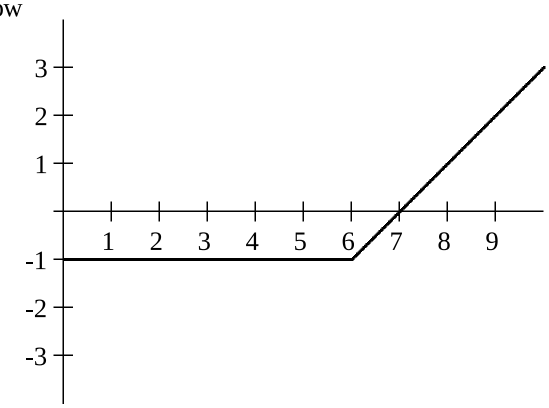
A. A hill or mountain.

B. The bottom of an ocean.

C. Level ground at sea level.

Answer: ____________________________________

18.(1 pt) Let $f(x)$ be the function whose graph is drawn below



and let

$$F(x) = \int_0^x f(t)\,dt$$

Fill in the table below

x	$F(x)$
0	____
1	____
2	____
3	____
4	____
5	____
6	____
7	____
8	____
9	____

The equation of the tangent line to the graph of $y = F(x)$ at $x = 8$ is
$y =$ ____________________