April 7, 2003 **Name** ______________________________

Math 112.01 – Quiz 8

1) Determine the form of the partial fraction decomposition of $\frac{x^9 + 2x^3 + x - 7}{(x^4 + x^3 + 2x^2)^2}$. Do NOT attempt to determine the constants.

Solution: First observe that this is an improper fraction so there will be a polynomial part (of degree 1) in the decomposition. A polynomial of degree 1 looks like $ax + b$. Next we need to factor the denominator completely which is $(x^2(x^2 + x + 2))^2 = x^4(x^2 + x + 2)^2$. Now we can write down the form of the decomposition:

$$\frac{x^9 + 2x^3 + x - 7}{(x^4 + x^3 + 2x^2)^2} = ax + b + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{Ex + F}{x^2 + x + 2} + \frac{Gx + I}{(x^2 + x + 2)^2}$$

2) Evaluate $\int \frac{x^3}{x^2 + 1} dx$

Solution: Notice that the integrand is an improper rational function. By long division, $\frac{x^3}{x^2+1} = x - \frac{x}{x^2+1}$. So,

$$\int \frac{x^3}{x^2 + 1} dx = \int (x - \frac{x}{x^2 + 1}) dx = \frac{x^2}{2} - \int \frac{x}{x^2 + 1} dx.$$

To evaluate the second integral, use the substitution $u = x^2 + 1$ and obtain

$$\int \frac{x^3}{x^2 + 1} dx = \frac{x^2}{2} - \frac{\ln(x^2 + 1)}{2} + C$$

3) Evaluate $\int \frac{dx}{x^4 + x^2}$

Solution: First find the partial fraction decomposition:

$$\frac{1}{x^4 + x^2} = \frac{1}{x^2(x^2 + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1}$$

Then determine the constants. They turn out to be $A = 0, B = 1, C = 0, D = -1$. Therefore

$$\int \frac{dx}{x^4 + x^2} = \int (\frac{1}{x^2} - \frac{1}{x^2 + 1}) dx = \frac{-1}{x} - \arctan(x) + C$$