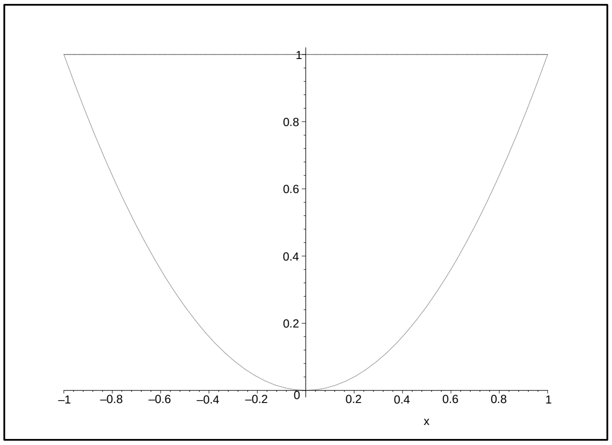
Feb 24, 2003 **Name** ____________________________

Math 112.01 – Quiz 5 with Solutions

1) Find the volume of the solid whose base is the parabolic region $\{(x, y) : x^2 \leq y \leq 1\}$ and the cross-sections perpendicular to y-axis are semicircles.

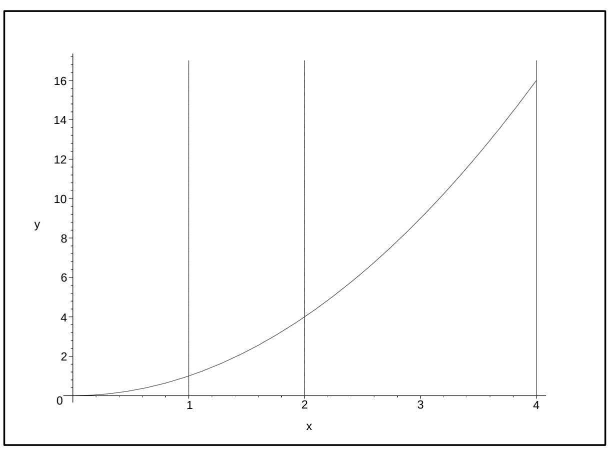
Solution: The region is



Since the cross sections are perpendicular to the y-axis, we need to integrate wrt y and from 0 to 1. A typical cross section y-units from the origin is a semicircle with diameter in the given region. The radius of the cross section is $x = \sqrt{y}$, therefore its area is $\frac{1}{2}\pi x^2 = \frac{1}{2}\pi y$. Therefore the integral which gives the volume is $\int_0^1 \frac{1}{2}\pi y dy = \frac{\pi}{4}$.

2) Let R be the region bounded by the $y = x^2$, $\quad y = 0$, $\quad x = 1$, and $x = 2$. Set up, but do NOT evaluate, an integral that would give the volume of the solid obtained by rotating the region R about $x = 4$. Sketch the region and show a typical strip. Specify which method you are using.

Solution: First sketch the region



Observe that in this case the method of cylindrical shells is more convenient. The integral for the shell method is

$$\int_1^2 2\pi(4 - x)x^2 dx$$

It is also possible to use the washer method in this problem but it requires two integrals (Why?). It would be

$$\int_0^1 \pi(3^2 - 2^2)dy + \int_1^4 \pi((4 - \sqrt{y})^2 - 2^2)dy$$

Verify that these two methods give the same answer!