Jan 31, 2003 **Name** ______________________________

Math 112.01 – Quiz 3 and Solutions

1) Let $f(x) = x^2 + 2x + 3$

i) Use $\sum$-notation to express the right Riemann sum for f on the interval $[0, 2]$ with regular partition having n subdivisions. Call this expression R_n.

ii) How does R_n compare to $\int_0^2 (x^2 + 2x + 3)dx$ (for any value of n)? Is it an underestimation, overestimation or could be either? Justify your answer.

iii) Compute $\lim_{n\to\infty} R_n$ to find the exact value of $\int_0^2 (x^2 + 2x + 3)dx$.

iv) Compute $\int_0^2 (x^2 + 2x + 3)dx$ using FTC (second version). Is it the same as what you found in part iii)?

Solutions: i)Here, $\Delta x = \frac{2-0}{n} = \frac{2}{n}$ and $c_k = x_k = \frac{2k}{n}$. So,

$$\begin{aligned} R_n &= \sum_{k=1}^{n} f(c_k)\Delta x_k = \sum_{k=1}^{n} f(\frac{2k}{n})\frac{2}{n} \\ &= \frac{2}{n}\sum_{k=1}^{n}\big((\frac{2k}{n})^2 + 2(\frac{2k}{n}) + 3\big) \\ &= \frac{8}{n^3}\sum_{k=1}^{n} k^2 + \frac{8}{n^2}\sum_{k=1}^{n} k + \frac{2}{n}\sum_{k=1}^{n} 3 \\ &= \frac{8n(n+1)(2n+1)}{6n^3} + \frac{8n(n+1)}{2n^2} + \frac{6n}{n} \\ &= \frac{8(n+1)(2n+1)}{6n^2} + \frac{8(n+1)}{2n} + 6 \end{aligned}$$

ii) $R_n \geq \int_0^2 (x^2 + 2x + 3)dx$ i.e., it is an overestimation for any value of n. This is because f is increasing on the interval [0,2]. (By a theorem we have, for an increasing function any right sum is an overestimation for the definite integral).

iii) $\lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{8(n+1)(2n+1)}{6n^2} + \lim_{n\to\infty} \frac{8(n+1)}{2n} + \lim_{n\to\infty} 6 = \frac{16}{6} + \frac{8}{2} + 6 = \frac{38}{3}$

iv)

$$\int_0^2 (x^2 + 2x + 3)dx = (\frac{x^3}{3} + x^2 + 3x)\Big|_0^2 = \frac{38}{6}$$

As expected, this answer is the same as what we found in part iii, because the definite integral is defined to be the limit of Riemann sums.