

Math 112.01 – Quiz 2 and Solutions

1) Evaluate $\int x \tan(x^2) dx$

Solution: Let $u = x^2$, then $du = 2x dx$ and $\frac{du}{2} = x dx$. Therefore, $\int x \tan(x^2) dx = \frac{1}{2} \int \tan(u) du$. To evaluate the latter integral we first write $\frac{1}{2} \int \tan(u) du = \frac{1}{2} \int \frac{\sin(u)}{\cos(u)} du$ then make another substitution: $v = \cos(u)$, so $dv = -\sin(u) du$. We then obtain

$$\frac{1}{2} \int \frac{\sin(u)}{\cos(u)} du = \frac{-1}{2} \int \frac{dv}{v} = \frac{-1}{2} \ln|v| + C = \frac{-1}{2} \ln|\cos(u)| + C = \frac{-1}{2} \ln|\cos(x^2)| + C$$

2) Evaluate $\int_e^{e^2} \frac{dx}{x \ln^2(x)}$

Solution: Let $u = \ln(x)$, then $du = \frac{dx}{x}$. Also note that the limits of integration should be changed to from $u(e) = \ln(e) = 1$ to $u(e^2) = \ln(e^2) = 2 \ln(e) = 2$. So

$$\int_e^{e^2} \frac{dx}{x \ln^2(x)} = \int_1^2 \frac{du}{u^2} = \frac{-1}{u} \Big|_1^2 = -\left(\frac{1}{2} - 1\right) = \frac{1}{2}$$