

## Math 112.01 – Quiz 1 and solutions

1) Find  $\frac{d}{dt} \int_{2t}^{t^2} \ln(u^2 + 1) du$

**Solution:** By the most general form of FTC Version I, we have

$$\frac{d}{dt} \int_{2t}^{t^2} \ln(u^2 + 1) du = \ln(t^4 + 1) \cdot 2t - \ln(4t^2 + 1) \cdot 2$$

2) Find  $\frac{d}{dx} \ln(x^2 + 1)$  then write the corresponding integration (anti-differentiation) formula.

**Solution:**  $\frac{d}{dx} \ln(x^2 + 1) = \frac{2x}{x^2+1}$ , therefore we obtain the integration formula

$$\int \frac{2x}{x^2+1} dx = \ln(x^2 + 1) + C$$

3) Check if the following formula is correct:  $\int e^x \sin(x) dx = \frac{e^x}{2} \cdot (\sin(x) - \cos(x)) + C$

**Solution:** Remember, to check whether a given integration formula is correct or not, you just need to differentiate the proposed antiderivative and see if you get the integrand. Here, differentiate the RHS (using the product rule) and simplify it. You will see that you get the integrand,  $e^x \sin(x)$ .

4) Given a continuous function  $f$  and a real number  $c$ , consider  $\int_c^x f(t) dt$  and  $\int f(x) dx$ . What exactly are these things? Are they the same thing? Are they related? How are they similar, how are they different?

**Solution:**  $\int f(x) dx$  is the family of all antiderivatives of  $f$  ( $F(x) + C$ ).  $\int_c^x f(t) dt$  is, however, a particular antiderivative, the one whose value at  $c$  is 0 (i.e. the value of  $C$  in  $F(x) + C$  is so chosen that  $F(c) + C = 0$ ).