Exam 3

Math 112: Calculus B Name: ____________________

100 points 04/25/2001

- You must show all work to receive full credit.

Evaluate the following:

1) (14 pts) $\int \sin^5(x)\,dx$

Solution:

$$\begin{aligned}
\int \sin^5(x)\,dx &= \int \sin^4(x)\sin(x)\,dx \\
&= \int (\sin^2(x))^2 \sin(x)\,dx \\
&= \int (1-\cos^2(x))^2 \sin(x)\,dx \quad u=\cos(x) \quad du=-\sin(x)\,dx \\
&= -\int (1-u^2)^2\,du \\
&= -\int (1-2u^2+u^4)\,du \\
&= -(u-\frac{2u^3}{3}+\frac{u^5}{5})+C \\
&= -(\cos(x)-\frac{2\cos^3(x)}{3}+\frac{\cos^5(x)}{5})+C
\end{aligned}$$

2) (14 pts) $\int (x^2 + x\sin(x))\,dx$

Solution:

$$\begin{aligned}
\int (x^2+x\sin(x))dx &= \int x^2\,dx + \int x\sin(x)\,dx \quad u=x \;\; du=dx \;\; v=-\cos(x) \;\; dv=\sin(x)\,dx \\
&= \frac{x^3}{3} - x\cos(x) + \int \cos(x)\,dx \\
&= \frac{x^3}{3} - x\cos(x) + \sin(x) + C
\end{aligned}$$

3) (14 pts) $\displaystyle\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$

Solution: $x = 2\tan,(\theta)\ \ dx = 2\sec^2(\theta),\ d\theta\ \ \sqrt{x^2+4} = 2\sec(\theta)$

$$\begin{aligned}
\int \frac{1}{x^2\sqrt{x^2+4}}\,dx &= \int \frac{2\sec^2(\theta)}{4\tan^2(\theta)(2)\sec(\theta)}\,d\theta \\
&= \frac{1}{4}\int \frac{\sec(\theta)}{\tan^2(\theta)}\,d\theta \\
&= \frac{1}{4}\int \frac{\cos(\theta)}{\sin^2(\theta)}\,d\theta \qquad u = \sin(\theta)\ \ du = \cos(\theta)\,d\theta \\
&= \frac{1}{4}\int \frac{1}{u^2}\,du \\
&= \frac{-1}{4u} + C \\
&= \frac{-1}{4\sin(\theta)} + C \\
&= \frac{-\sqrt{x^2+4}}{4x} + C
\end{aligned}$$

4) (14 pts) $\displaystyle\int \frac{x^2+x-3}{(x-2)(x^2-1)}\,dx$

Solution: $\dfrac{x^2+x-3}{(x-2)(x^2-1)} = \dfrac{A}{x-2} + \dfrac{B}{x+1} + \dfrac{C}{x-1} \Rightarrow A = 1,\ B = \dfrac{-1}{2},\ C = \dfrac{1}{2}$

$$\begin{aligned}
\int \frac{x^2+x-3}{(x-2)(x^2-1)}\,dx &= \int \frac{1}{x-2}\,dx - \frac{1}{2}\int \frac{1}{x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx \\
&= \ln|x-2| - \frac{1}{2}\ln|x+1| + \frac{1}{2}\ln|x-1| + C
\end{aligned}$$

5) (14 pts) $\int x\tan^{-1}(x)\,dx$

Solution: $u=\tan^{-1}(x)$ $du=\frac{1}{1+x^2}\,dx$ $v=\frac{x^2}{2}$ $dv=x\,dx$ and using long division

$$\begin{aligned}\int x\tan^{-1}(x)\,dx &= \frac{x^2\tan^{-1}(x)}{2}-\frac{1}{2}\int\frac{x^2}{1+x^2}\,dx\\ &= \frac{x^2\tan^{-1}(x)}{2}-\frac{1}{2}[\int 1\,dx-\int\frac{1}{1+x^2}\,dx]\\ &= \frac{x^2\tan^{-1}(x)}{2}-\frac{x}{2}+\frac{\tan^{-1}(x)}{2}+C\end{aligned}$$

6) (15 pts) $\int_{-2}^{2}\frac{1}{2x+3}\,dx$

Solution:

$$\begin{aligned}\int_{-2}^{2}\frac{1}{2x+3}\,dx &= \int_{-2}^{\frac{-3}{2}}\frac{1}{2x+3}\,dx+\int_{\frac{-3}{2}}^{2}\frac{1}{2x+3}\,dx\\ &= \lim_{b\to\frac{-3}{2}}\int_{-2}^{b}\frac{1}{2x+3}\,dx+\lim_{a\to\frac{-3}{2}}\int_{a}^{2}\frac{1}{2x+3}\,dx\\ &= \lim_{b\to\frac{-3}{2}}\frac{\ln|2x+3|}{2}\Big|_{-2}^{b}+\lim_{a\to\frac{-3}{2}}\frac{\ln|2x+3|}{2}\Big|_{a}^{2}\\ &= \lim_{b\to\frac{-3}{2}}[\frac{\ln|2b+3|}{2}-\frac{\ln|1|}{2}]+\lim_{a\to\frac{-3}{2}}[\frac{\ln|2a+3|}{2}-\frac{\ln|7|}{2}\end{aligned}$$

Diverges

7) (15 pts) $\int_{-\infty}^{\infty} 2xe^{-x^2}\,dx$

Solution: $u = -x^2 \;\; du = -2x\,dx$

$$\begin{aligned}
\int_{-\infty}^{\infty} 2xe^{-x^2}\,dx &= \int_{-\infty}^{0} 2xe^{-x^2}\,dx + \int_{0}^{\infty} 2xe^{-x^2}\,dx \\
&= \lim_{t\to-\infty}\int_{t}^{0} 2xe^{-x^2}\,dx + \lim_{s\to-\infty}\int_{0}^{s} 2xe^{-x^2}\,dx \\
&= \lim_{t\to-\infty}\int_{t}^{0} e^{-u}\,du + \lim_{s\to-\infty}\int_{0}^{s} e^{-u}\,du \\
&= \lim_{t\to-\infty} -e^{-u}|_{t}^{0} + \lim_{s\to-\infty} -e^{-u}|_{0}^{s} \\
&= \lim_{t\to-\infty} -e^{-x^2}|_{t}^{0} + \lim_{s\to-\infty} -e^{-x^2}|_{0}^{s} \\
&= \lim_{t\to-\infty}[-1+e^{-t^2}] + \lim_{s\to-\infty}[-e^{-s^2}+1] \\
&= -1+1 \\
&= 0
\end{aligned}$$

Converge