Exam 2

Math 112: Calculus B Name: ____________

100 points 03/28/2001

• You must show all work to receive full credit.

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1) (10 points each) Evaluate the following:

a) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

Solution:

$$\begin{aligned} u &= \sqrt{x} \\ du &= \frac{1}{2\sqrt{x}}\,dx \end{aligned}$$

$$\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx = 2\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = 2\int e^u\,du = 2e^u + C = 2e^{\sqrt{x}} + C$$

b) $\int \frac{\sec^2(x)}{1+\tan(x)}\,dx$

Solution:

$$\begin{aligned} u &= 1+\tan(x) \\ du &= \sec^2(x)\,dx \end{aligned}$$

$$\int \frac{\sec^2(x)}{1+\tan(x)}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|1+\tan(x)| + C$$

c) $\int (\ln(x))^2\,dx$

Solution:

$$\begin{aligned} u &= (ln(x))^2 \\ du &= \frac{2\ln(x)}{x}\,dx \end{aligned}$$

$$\begin{aligned} v &= x \\ dv &= dx \end{aligned}$$

$$\int (\ln(x))^2\,dx = x(\ln(x))^2 - 2\int \ln(x)\,dx$$

$$\begin{aligned} u &= ln(x) \\ du &= \frac{1}{x}\,dx \end{aligned}$$

$$\begin{aligned} v &= x \\ dv &= dx \end{aligned}$$

$$\int (\ln(x))^2\,dx = x(\ln(x))^2 - 2x\ln(x) + 2\int\,dx = x(\ln(x))^2 - 2x\ln(x) + 2x + C$$

d) $\int e^x \cos(x)\,dx$

Solution:

$$\begin{aligned} u &= e^x \\ du &= e^x\,dx \end{aligned}$$

$$\begin{aligned} v &= \sin(x) \\ dv &= \cos(x)dx \end{aligned}$$

$$\int e^x\cos(x)\,dx = e^x\sin(x) - \int e^x\sin(x)\,dx$$

$$\begin{aligned} u &= e^x \\ du &= e^x\,dx \end{aligned}$$

$$\begin{aligned} v &= -\cos(x) \\ dv &= \sin(x)dx \end{aligned}$$

$$\begin{aligned} \int e^x\cos(x)\,dx &= e^x\sin(x) + e^x\cos(x) - \int e^x\cos(x)\,dx \\ 2\int e^x\cos(x)\,dx &= e^x\sin(x) + e^x\cos(x) \\ \int e^x\cos(x)\,dx &= \frac{e^x\sin(x) + e^x\cos(x)}{2} + C \end{aligned}$$

2) (15 points) Find the area of the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

Solution: $x = y + 1$, $x = \dfrac{y^2 - 6}{2}$

$$y + 1 = \frac{y^2 - 6}{2} \Rightarrow y = 4, y = 2$$

$$\int_{-2}^{4} [(y+1) - (\frac{y^2 - 6}{2}] \, dy = \int_{-2}^{4} [\frac{-1}{y^2} + y + 4] \, dy = (\frac{-y^3}{6} + \frac{y^2}{2} + 4y)|_{-2}^{4} = 18$$

3) (15 points) Find the volume of the solid of revolution generated by revolving the region bounded by $y = \sqrt{x}$, $y = 9$, and $x = 0$. about the line $x = -1$.

Solution:

$$\pi \int_0^9 [(y^2 + 1)^2 - 1^2] \, dy = \pi \int_0^9 [y^4 + 2y^2] \, dy = \pi(\frac{y^5}{5} + \frac{2y^3}{3})|_0^9 = 12295.8\pi$$

4) (15 points) Find the arclength of the curve $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}$ from $x = 2$ to $x = 4$.

Solution: $y' = (\dfrac{1}{3})(\dfrac{3}{2})\sqrt{x^2+2}(2x) = x\sqrt{x^2+2}$

$1 + (y')^2 = 1 + x^2(x^2+2) = (x^2+1)^2$

$$\begin{aligned}
\int_2^4 \sqrt{(x^2+1)^2}\,dx &= \int_2^4 (x^2+1)\,dx \\
&= \frac{x^3}{3} + x|_2^4 \\
&= 20.\overline{6}
\end{aligned}$$

5) (15 points) A spring whose natural length is 4 feet is stretched to a length of 8 feet when 2 pounds of force is applied. If 8 foot-pounds of work is expended on the spring (starting at its natural length), how far is it stretched.

Solution: $2 = 4k \Rightarrow k = \dfrac{1}{2}$

$$\begin{aligned}
\int_0^a \frac{x}{2}\,dx &= 8 \\
\frac{x^2}{2}|_0^a &= 8 \\
\frac{a^2}{2} &= 8
\end{aligned}$$

$a = \sqrt{32}$