Exam 2

Math 112: Calculus B Name: ____________________

100 points 10/21/2001

• You must show all work to receive full credit.

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1) (10 points each) Evaluate the following:

a) $\displaystyle\int \cot(x)\,dx$

Solution: $u = \sin(x)$, $du = \cos(x)\,dx$

$$\begin{aligned}\int \cot(x)\,dx &= \int \frac{\cos(x)}{\sin(x)}\,dx \\ &= \int \frac{1}{u}\,du \\ &= \ln|u| + C \\ &= \ln|\sin(x)| + C\end{aligned}$$

b) $\displaystyle\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx$

Solution: $u = 1 + \sqrt{x}$, $du = \dfrac{1}{s\sqrt{x}}\,dx$

$$\begin{aligned}\int \frac{1}{\sqrt{x}(1+\sqrt{x})}\,dx &= 2\int \frac{1}{u}\,du \\ &= 2\ln|u| + C \\ &= 2\ln|1+\sqrt{x}| + C\end{aligned}$$

c) $\displaystyle\int \frac{x}{\sqrt{4-x}}\,dx$

Solution: $u = 4-x,\;\; du = -dx\;\; x = 4-u$

$$\begin{aligned}
\int \frac{x}{\sqrt{4-x}}\,dx &= -\int \frac{4-u}{\sqrt{u}}\,du \\
&= -\int (4u^{\frac{-1}{2}} - u^{\frac{1}{2}})\,du \\
&= -(8u^{\frac{1}{2}} - \frac{2}{3}u^{\frac{3}{2}}) + C \\
&= -8\sqrt{4-x} - \frac{2}{3}\sqrt{(4-x)^3} + C
\end{aligned}$$

d) $\displaystyle\int \frac{2x}{e^x}\,dx$

Solution: $u = 2x,\;\; du = 2\,dx,\;\; dv = e^{-x}\,dx,\;\; v = -e^{-x}$

$$\begin{aligned}
\int \frac{2x}{e^x}\,dx &= -2xe^{-x} + 2\int e^{-x}\,dx \\
&= -2xe^{-x} - 2e^{-x} + C
\end{aligned}$$

e) $\displaystyle\int \frac{(\ln(x))^2}{x}\,dx$

Solution: $u = \ln(x), \quad du = \dfrac{1}{x}\,dx$

$$\begin{aligned} \int \frac{(\ln(x))^2}{x}\,dx &= \int u^2\,du \\ &= \frac{u^3}{3} + C \\ &= \frac{(\ln(x))^3}{3} + C \end{aligned}$$

2) (10 points) A 20-foot chain weighing 8 pounds per foot is lying on the ground. How much work is required to raise the chain 20-feet so that it is fully extended vertically.

Solution: Weight: $8\,Deltay$, Distance Lifted : y

$$\int_0^{20} 8y\,dy = 4y^2|_0^{20} = 1600$$

3) (10 points) Find the volume of the solid of revolution generated by revolving the region bounded by $y = 6 - x^2$, and $y = 2$ about the line $y = 2$.

Solution: $6 - x^2 = 2 \Rightarrow x = \pm 2$, radius $= 6 - x^2 - 2 = 4 - x^2$

$$\begin{aligned} \pi \int_{-2}^{2} (4 - x^2)^2 \, dx &= \pi \int_{-2}^{2} (16 - 8x^2 + x^4) \, dx \\ &= \pi (16x - \frac{8x^3}{3} + \frac{x^5}{5})|_{-2}^{2} \\ &\approx 34.13\pi \end{aligned}$$

4) (10 points) The base of a certain solid has the shape of the region bounded by $y = x^2$, $y = -x^2 - 1$, $x = 0$, and $x = 2$. Determine the volume of the solid if vertical cross sections perpendicular to the x-axis are semicircles.

Solution: Diameter $= x^2 - (-x^2 - 1) \Rightarrow$ Radius $= \dfrac{2x^2 + 1}{2} \Rightarrow A(x) = \dfrac{\pi}{2} \left(\dfrac{2x^2 + 1}{2} \right) = \dfrac{\pi}{8} (4x^4 + 4x^2 + 1)$

$$\frac{pi}{8} \int_{0}^{2} (4x^4 + 4x^2 + 1) \, dx = \frac{pi}{8} \left(\frac{4x^5}{5} + \frac{4x^3}{3} + x \right)|_{0}^{2} \approx \frac{38.27\pi}{8}$$

5) (10 points) Given the initial value problem

$$\begin{aligned} \frac{dy}{dx} &= x - y^2 \\ y(0) &= 1 \end{aligned}$$

approximate $y(1)$ using Euler's method with step size equal to 0.25.
Solution: $x_0 = 0, \;\; y_0 = 1$
$x_1 = 0.25, \;\; y_1 = 1 + 0.25(0 - 1^2) = 0.75$
$x_2 = 0.5, \;\; y_1 = 0.75 + 0.25(0.25 - (0.75)^2) = 0.671875$
$x_3 = 0.75, \;\; y_1 = 0.671875 + 0.25(0.5 - (0.671875)^2) = 0.684021$
$x_4 = 1, \;\; y_1 = 0.684021 + 0.25(0.75 - (0.684021)^2) = 0.75455$

6) (10 points) Given the sloe field for the differential equation $\frac{dy}{dx} = x - y^2$, draw an approximate solution the initial value problem given in question (5).
Solution: