Exam 1

Math 112: Calculus B Name: ________________

100 points 09/07/2001

• You must show all work to receive full credit.

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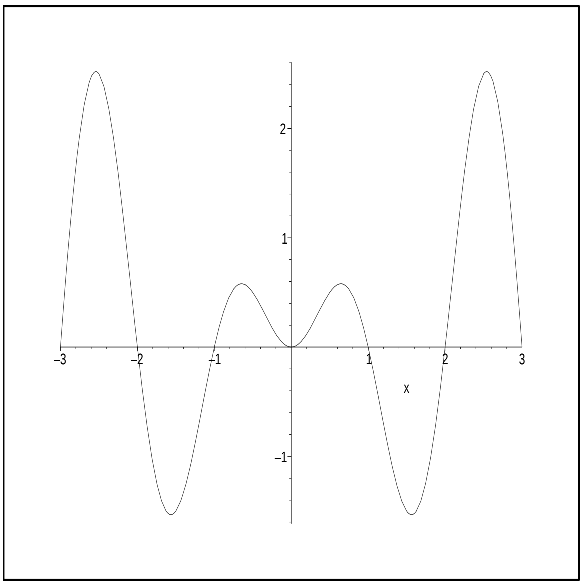
1) (5 pts) Let $A_f(x) = \int_0^x f(x)\,dx$ where $f(x)$ is the function graphed below on the interval [-3,3]. Identify the location, by indicating the x-coordinate, of all local maxima and all local minima for $A_f(x)$ on the interval (-3, 3).



Solution:

local maximums at $x = -2$, $x = 1$

local minimums at $x - 1$, $x = 2$

2) (9pts) Given that $f(x) = 3x^2 - \sec^2(x)$ determine the formula for the function $A_f(x) = \int_{2\pi}^x f(x)\,dx$.

Solution: An anti derivative is given by $F(x) = x^3 = \tan(x)$

Since $F(2\pi) = (2\pi)^3 - \tan(2\pi) = 8\pi^3$ we have that $A_f(x) = x^3 = \tan(x) - 8\pi^3$

3) (8 pts each) Use the Fundamental Theorem of Calculus to evaluate the following:

a) $\frac{d}{dx}\int_0^x \tan(\frac{\pi e^t}{4})\, dt$

Solution: $\tan(\frac{\pi e^x}{4})$

b) $\int_0^x \frac{d}{dt}\tan(\frac{\pi e^t}{4})\, dt$

Solution: $\tan(\frac{\pi e^x}{4}) - \tan(\frac{\pi}{4}) = \tan(\frac{\pi e^x}{4}) - 1$

c) $\frac{d}{dx}\int_{\sqrt{x}}^{x^2} \ln(\sec(t))\, dt$

Solution: $\frac{d}{dx}[-\int_a^{\sqrt{x}} \ln(\sec(t))\, dt + \int_a^{x^2} \ln(\sec(t))\, dt]$

$\ln(\sec(x^2))(2x) - \ln(\sec(\sqrt{x})(\frac{1}{2\sqrt{x}})$

$(2x)\ln(\sec(x^2)) - (\frac{\ln(\sec(\sqrt{x})}{2\sqrt{x}})$

d) $\int_{\sqrt{x}}^{x^2} \frac{d}{dt}\ln(\sec(t))\, dt$

Solution: $\ln(\sec(x^2)) - \ln(\sec(\sqrt{x}))$

4) (9 pts each) Evaluate the following:

a) $\int (x - \frac{1}{x})\, dx$

Solution: $\frac{x^2}{2} - \ln|x| + C$

b) $\int (3x-1)^2\, dx$

Solution:

$$\begin{aligned} u &= 3x - 1 \\ du &= 3\,dx \end{aligned}$$

$$\int (3x-1)^2\, dx = \frac{1}{3}\int 3(3x-1)^2\, dx = \frac{1}{3}\int u^2\, du = \frac{1}{3}\frac{u^3}{3} + C = \frac{(3x-1)^3}{9} + C$$

c) $\int \frac{\sin(x)}{\cos^5(x)}\, dx$

Solution:

$$\begin{aligned} u &= \cos(x) \\ du &= -\sin(x)\, dx \end{aligned}$$

$$\int \frac{\sin(x)}{\cos^5(x)}\, dx = -\int \frac{1}{u^5}\, du = -\frac{u^{-4}}{-4} + C = \frac{1}{4\cos^4(x)} + C$$

d) $\int_{-3}^{3} x\sqrt{9-x^2}\, dx$

Solution:

$$\begin{aligned} u &= 9 - x^2 \\ du &= -2x\, dx \end{aligned}$$

$$\int_{-3}^{3} x\sqrt{9-x^2}\, dx = \frac{-1}{2}\int_{-3}^{3} \sqrt{u}\, du = (\frac{-1}{2})(\frac{2}{3})u^{\frac{3}{2}}|_{-3}^{3} = 0$$

5) (8 pts) Write $\int_0^4 x\sin(x)\,dx$ as the limit of a Riemann Sum. You do not have to evaluate the sum or evaluate the limit.

Solution: $f(x) = x\sin(x)$, $a = 0$, $b = 4$, $\Delta x = \frac{4}{n}$

Right Sum:

$$\begin{aligned}\lim_{n\to\infty}\sum_{i=1}^{n} f(\frac{4i}{n})(\frac{4}{n}) &= \lim_{n\to\infty}\sum_{i=1}^{n}(\frac{4i}{n})\sin(\frac{4i}{n})(\frac{4}{n}) \\ &= \lim_{n\to\infty}\sum_{i=1}^{n}(\frac{16i}{n^2})\sin(\frac{4i}{n})\end{aligned}$$

Left Sum:

$$\begin{aligned}\lim_{n\to\infty}\sum_{i=1}^{n} f(\frac{4(i-1)}{n})(\frac{4}{n}) &= \lim_{n\to\infty}\sum_{i=1}^{n}(\frac{4(i-1)}{n})\sin(\frac{4(i-1)}{n})(\frac{4}{n}) \\ &= \lim_{n\to\infty}\sum_{i=1}^{n}(\frac{16(i-1)}{n^2})\sin(\frac{4i}{n})\end{aligned}$$

6) (10 pts) Approximate $\int_{-1}^{1} xe^{x^2}\,dx$ with the Trapezoidal Rule using four subintervals.

Solution: $f(x) = xe^{x^2}$, $\Delta x = \dfrac{1}{2}$

$$\begin{aligned}\int_{-1}^{1} xe^{x^2}\,dx &\approx \sum_{i=1}^{4}\frac{f(-1+\frac{i-1}{2})+f(-1+\frac{i}{2})}{2}\frac{1}{2} \\ &= \sum_{i=1}^{4}\frac{1}{4}f(-1+\frac{i-1}{2})+f(-1+\frac{i}{2}) \\ &= \frac{1}{4}[(f(-1)+f(-0.5))+(f(-0.5)+f(0))+(f(0)+f(0.5))+(f(0.5)+f(1))] \\ &= .8481\end{aligned}$$