Practice on Differential Equations

1)Find the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{\ln(x)}{xy + xy^3}$

2) Find the particular solutions of the following initial value problems:

$a)\begin{cases} \frac{dy}{dx} = (x-4)e^{-2y} \\ y(4) = \ln(4) \end{cases}$ $\qquad$ $b)\begin{cases} \frac{dy}{dx} + y\cos(x) = 3\cos(x) \\ y(0) = 1 \end{cases}$

FOR THE FOLLOWING PROBLEMS YOU MAY NEED TO USE MAPLE

3) Hot coffee in a 70-degree room cools at a rate proportional to the difference between the coffee temperature and the room temperature. (This is known as Newton's law of cooling). Letting $y(t)$ represent the temperature of the coffee at time t, we can write the statement mathematically as:

$$\frac{dy}{dt} = k(y(t) - 70)$$

Suppose you measured the temperature of the coffee at a certain time (let's call this time $t = 0$), and you found it to be 190 degrees. Suppose also that you discovered the coffee's temperature, at time $t = 0$, is dropping at a rate of 12 degrees per minute.
a) Use the given information to compute the proportionality constant k.

b) Determine the function $y(t)$ (i.e. solve the differential equation).

c) How long will it take for the coffee to reach the temperature of 140 degrees?

d) How long will it take for the coffee to cool from 140 degrees to 90 degrees?

e) In parts c) and d) you found the amounts of time needed for 50-degree drops in temperature. Are they equal? If not, explain why.

f) Use Maple to produce a plot of the slope field of this differential equation. Does it provide an explanation for part e?

4) Student loans, mortgage loans and retirement accounts all require similar and surprisingly complex mathematics. Most customers are unprepared to check the bank's calculations. The mathematics involved is partly continuous (quantities changing all the time) and partly discrete (quantities changing at fixed intervals). Appropriately, our tools will be partly continuous partly discrete.
Suppose you borrowed \$ 10,000 at 7% interest rate. Without payments, the amount owed, or the principal L, increases at a rate proportional to itself: $L'(t) = .07L(t)$ (Does this type of DE look familiar?) Although the model is continuous, you actually expect to make payments discretely (once a month). We can model the situation continuously however, without loosing much accuracy. Suppose you will be making payments of \$150 month. That is \$1800 per year. Therefore we have, $L' = 0.07L - 1800$ (since we are measuring t in years). The initial loan was for \$ 10,000 meaning $L(0) = 10,000$.

a) Find a formula for the principal (money owed) at any time t.

b) How long does it take to pay all the money back?

c) Generalize this situation for any loan amount A and any interest rate r.