Series

1) Consider the series $\sum_{k=0}^{\infty}(-1)^k$

i) Write out the first five partial sums of this series.

ii) Find a general formula for the partial sums. Does it converge? Does the series converge?

2) Consider the series $\sum_{k=1}^{\infty}\ln(\frac{k+1}{k})$.

i) Does the n-th term test say anything about this series?

ii) Show that this is a telescoping series and find a formula for the partial sums. Does the series converge?

3) Consider the series $\sum_{n=0}^{\infty}(1-\frac{1}{n})^n$. Does the n-th term test apply to this series? Does the sequence of general terms converge? Does the series converge?

4) Suppose that $\sum a_n$ converges. Show that $\sum \cos(a_n)$ must diverge.

5)For the following series determine whether they converge or not. In case of convergence, find the exact sum.

i) $\displaystyle\sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n}$ ii) $\displaystyle\sum_{n=1}^{\infty} (\frac{-5}{9})^n$ iii) $\displaystyle\sum_{n=0}^{\infty} (\frac{\pi}{e})^n$

6) Write $0.\bar{3}$ as a geometric series and find its sum

7) Determine whether the following statements are true or false.

i) If $\lim a_n = 0$ then $\sum a_n$ converges.

ii) If $\sum a_n$ converges then $\lim a_n = 0$

iii) If $\sum a_n$ does not converge then $\lim a_n \neq 0$

iv) If $\sum a_n$ does not converge then who knows what the $\lim a_n$ is.

v) A geometric series $\sum r^n$ is always convergent with sum $\frac{1}{1-r}$