Sequences

Determine whether the following sequences converge. In case of convergence, find the limit.

1) $a_n = \dfrac{n}{e^n}$

2) $a_n = 1 + (-1)^n$

3) $a_n = \dfrac{\sin(n)}{\ln(n)}$

4) $\sqrt{3}^n$

5)Determine whether the following sequences are increasing, decreasing or not monotone.

$i) a_n = \dfrac{1}{3n+5}$ $\quad ii) a_n = \dfrac{n-2}{n+2}$ $\quad iii) a_n = 3 + \dfrac{(-1)^n}{n}$ $\quad iv) a_n = \dfrac{n}{n^2+n-1}$

Directions: Determine whether the following statements are true or false. Justify your answer. If the answer is false, provide a counterexample.

1) If $\lim |a_n| = L$ then $\lim |a_n| = \pm L$

2) If $\lim a_n = L$ then $\lim |a_n| = \pm |L|$

3) Every bounded sequence is convergent.

4) Every convergent sequence is bounded.

5) Every bounded monotone sequence is convergent.

A Bonus Problem: If you provide a complete and rigorous solution to this problem by Friday, you will get 5 extra points. Let a_n be defined as follows:

$a_1 = 1$, $a_{n+1} = \sqrt{3 + a_n}$ for $n \geq 1$. Show that

i) a_n is bounded from above by 3 (use induction)

ii) a_n is increasing (can use induction again)

iii) Conclude that a_n is convergent. Find its limit.