July 23, 2002 **Name** ______________________________

Math. 152 – Midterm 1

Directions:

- Show your work. The way you derive your answer is more important than the answer itself.
- Your answers and claims must follow from your work. You may loose points for irrelevant or incorrect statements you put on the paper.
- Always give exact answers, not decimal point approximations. For example if the answer is $\sqrt{2}$ don't give 1.141... You do not need any calculators for this exam.

1) (16 points) The sum $$\sum_{k=1}^{n}\left(\sqrt{4-(\frac{\sqrt{2}k}{n})^2}-\frac{\sqrt{2}k}{n}\right)\frac{\sqrt{2}}{n}$$
is a right-Riemann sum of a certain function $f(x)$ on the interval $[0,\sqrt{2}]$ corresponding to the equal-length partition with n equal parts.
a) What is the function $f(x)$?
b) What is the limit of these Riemann sums as $n \to \infty$?
c) The result in part b) should be a definite integral. Sketch the region whose are is given by this definite integral. (Hint: Consider the function $f(x)$ as a difference of two other functions and the region as the region btw two functions)
d) Find the value of this integral by computing the area of the region by an elementary formula. (You won't be able to evaluate the integral directly)

2)(21 points) Evaluate the following indefinite integrals

$$i)\int \frac{x^2+1}{\sqrt{x}}dx \qquad ii)\int x^2\sqrt{3x+4}dx \qquad iii)\int \frac{dx}{x^2+4x+6}$$

3)(21 points) Evaluate the following definite integrals

$$i)\int_0^{\sqrt{2}} \frac{e^{\sqrt{2}}}{\sqrt{2}}dx \text{ (Hint: much easier than you think!)} \quad ii)\int_0^{\frac{\pi}{4}} |\frac{\sqrt{3}}{2}-\sin 2x|dx \qquad iii)\int_{\ln(\frac{1}{\sqrt{2}})}^{0} \frac{e^x dx}{\sqrt{1-e^{2x}}}$$

4)(14 points) i)State the generalized form of the first version of the FTC

ii)Use i) to find the derivative of the function $F(x)=\displaystyle\int_{2^x}^{e} \log_2 t dt$

5)(14 points) Sketch the region between the curves $y = \sqrt{x}$ and $y = x^2$ on the interval [0,2]. Then find its area.

6)(21 points) i) State the base-change formula for logarithms.

ii) Find the derivatives of the each of the following functions.

$$a) \log_3 (4^x + 1) \qquad b) x^x \qquad c) x^{x^x} \text{ (Hint } x^{x^x} \text{ means } x^{(x^x)} \text{ and also use part b)}$$

7)(20 points) i)Show that the function $f(x) = \frac{2^x-1}{2^x+1}$ has an inverse on the entire real line.
ii) Find a formula for the inverse function.
iii) Find the domain of the inverse function.
iv) Find $\lim_{x \to \infty} f(x)$, and $\lim_{x \to -\infty} f(x)$. Let $a = f^{-1}(0)$. Find a. What should $f(a)$ be? Check that it is indeed equal to what it is supposed to be.

8)(10 points) Kerry and Henry are debating over $\int x^2 \sin x dx$. Kerry thinks that the answer is $x^2 \sin x + 2x \cos x - 2 \sin x + C$. Henry does not agree, his answer is $-x^2 \cos x + 2x \sin x + 2 \cos x + C$. Who, if any, is right? Justify your claim.

9)(13 points)Determine whether the following statements are True or False. No explanation is necessary.

i) If a function is decreasing over an interval [a,b], then its upper sum is the same as its right sum.

ii) An arbitrary Riemann sum for a function $f(x)$ over an interval [a,b] is always greater than the left sum for $f(x)$ on [a,b].

iii)Mid-point sum is a special case of Riemann sum.

iv) $\ln a - \ln b = \frac{\ln a}{\ln b}$ for any $a, b > 0$.

v) $\log_\pi x$ is the inverse function for π^x.

vi) $a^x \cdot b^{x^2} = (a \cdot b^x)^x$

vii) Calculus is so beautiful that it would still be a worthwhile subject to learn even if it was not as useful as it is.