PRACTICE1 **Math 152 - Section 99999 Summer 2001**

Homework Set Hmwk7 due 6/2/09 at 6:00 AM

This set covers sections 7.7, 8.1, 8.2 and 8.3 of the text.

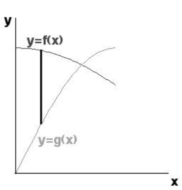
Problems 2, 3, 4, 5, and 6 in this set refer to figures/pictures. These will be difficult if not impossible to see in the hard copy (because they are scaled down from low resolution images). To get a clear printout of the pictures, go to the online version of each problem and click on the thumbnail of the picture. A full scale version of the picture will open up in a new window, which you can then print from your browser, one picture at a time.

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

1.(1 pt) Consider the following integrals. Label each as "P", "C", "D", according as the integral is proper, improper but convergent, or improper and divergent.

__ 1. $\int_{-\infty}^{\infty} \sin(3x)\,dx$
__ 2. $\int_{-10\pi}^{30\pi} \sin(x)\tan^{-1}(x)\,dx$
__ 3. $\int_{-\pi/6}^{13\pi/2} \tan^2(3x)\,dx$
__ 4. $\int_1^{\infty} se^{-3s^2}\,ds$
__ 5. $\int_3^{\infty} \frac{1}{\sqrt{t^2-9}}\,dt$
__ 6. $\int_{-\infty}^{\infty} \frac{x}{x^2+10}\,dx$
__ 7. $\int_6^{13} \ln(x-6)\,dx$
__ 8. $\int_0^{13} \frac{1}{\sqrt[3]{x-6}}\,dx$

This problem covers section 7.7 of the text. For further practice, you should do 5-10 odd numbered problems from among problems 1-59 on pages 486-487.

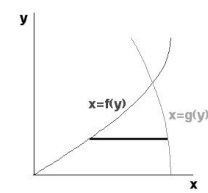


2.(1 pt)
Consider the blue vertical line shown above (click on graph for better view) connecting the graphs $y = g(x) = \sin(4x)$ and $y = f(x) = \cos(4x)$.
Referring to this blue line, match the statements below about rotating this line with the corresponding statements about the result obtained.

__ 1. The result of rotating the line about the x-axis is
__ 2. The result of rotating the line about the y-axis is
__ 3. The result of rotating the line about the line $y = 1$ is
__ 4. The result of rotating the line about the line $x = -2$ is
__ 5. The result of rotating the line about the line $x = \pi$ is
__ 6. The result of rotating the line about the line $y = -2$ is
__ 7. The result of rotating the line about the line $y = \pi$
__ 8. The result of rotating the line about the line $y = -\pi$

A. a cylinder of radius $\pi - x$ and height $\cos(4x) - \sin(4x)$
B. an annulus with inner radius $\pi + \sin(4x)$ and outer radius $\pi + \cos(4x)$
C. an annulus with inner radius $\sin(4x)$ and outer radius $\cos(4x)$
D. an annulus with inner radius $1 - \cos(4x)$ and outer radius $1 - \sin(4x)$ is
E. a cylinder of radius x and height $\cos(4x) - \sin(4x)$
F. an annulus with inner radius $2 + \sin(4x)$ and outer radius $2 + \cos(4x)$
G. an annulus with inner radius $\pi - \cos(4x)$ and outer radius $\pi - \sin(4x)$
H. a cylinder of radius $x + 2$ and height $\cos(4x) - \sin(4x)$

This problem is intended to help you set up integrals for volumes of solids of revolution, discussed in sections 8.1 and 8.2 of the text.

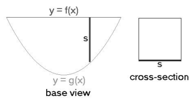


3.(1 pt)
Consider the blue horizontal line shown above (click on graph for better view) connecting the graphs $x = f(y) = \sin(1y)$ and $x = g(y) = \cos(1y)$.
Referring to this blue line, match the statements below about rotating this line with the corresponding statements about the result obtained.

__ 1. The result of rotating the line about the x-axis is
__ 2. The result of rotating the line about the y-axis is
__ 3. The result of rotating the line about the line $y = 1$ is
__ 4. The result of rotating the line about the line $x = -2$ is
__ 5. The result of rotating the line about the line $x = \pi$ is
__ 6. The result of rotating the line about the line $y = -2$ is
__ 7. The result of rotating the line about the line $y = \pi$
__ 8. The result of rotating the line about the line $y = -\pi$

A. an annulus with inner radius $\sin(1y)$ and outer radius $\cos(1y)$
B. a cylinder of radius $1 - y$ and height $\cos(1y) - \sin(1y)$
C. a cylinder of radius y and height $\cos(1y) - \sin(1y)$
D. a cylinder of radius $2 + y$ and height $\cos(1y) - \sin(1y)$
E. a cylinder of radius $\pi + y$ and height $\cos(1y) - \sin(1y)$
F. an annulus with inner radius $\pi - \cos(1y)$ and outer radius $\pi - \sin(1y)$ is
G. a cylinder of radius $\pi - y$ and height $\cos(1y) - \sin(1y)$
H. an annulus with inner radius $2 + \sin(1y)$ and outer radius $2 + \cos(1y)$

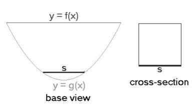
This problem is intended to help you set up integrals for volumes of solids of revolution, discussed in sections 8.1 and 8.2 of the text.



4.(1 pt)
The base of a certain solid is the area bounded above by the graph of $y = f(x) = 9$ and below by the graph of $y = g(x) = 36x^2$. Cross-sections perpendicular to the x-axis are squares. (See picture above, click for a better view.)
Use the formula

$$V = \int_a^b A(x)\,dx$$

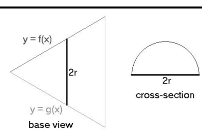
to find the volume of the formula.
Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.
The lower limit of integration is a = ______
The upper limit of integration is b = ______
The side s of the square cross-section is the following function of x: ______
$A(x)$= ______
Thus the volume of the solid is V = ______
This problem is similar to problems 29-34 of section 8.1 of the text.



5.(1 pt)
The base of a certain solid is the area bounded above by the graph of $y = f(x) = 4$ and below by the graph of $y = g(x) = 36x^2$. Cross-sections perpendicular to the y-axis are squares. (See picture above, click for a better view.)
Use the formula

$$V = \int_a^b A(y)\,dy$$

to find the volume of the formula.
Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.
The lower limit of integration is a = ______
The upper limit of integration is b = ______
The side s of the square cross-section is the following function of y: ______
$A(y)$= ______
Thus the volume of the solid is V = ______
This problem is similar to problems 29-34 of section 8.1 of the text.



6.(1 pt)
The base of a certain solid is an equilateral triangle with altitude 6. Cross-sections perpendicular to the altitude are semicircles. Find the volume of the solid, using the formula

$$V = \int_a^b A(x)\,dx$$

applied to the picture shown above (click for a better view), with the left vertex of the triangle at the origin and the given altitude along the x-axis.
Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.
The lower limit of integration is a = ______
The upper limit of integration is b = ______
The diameter $2r$ of the semicircular cross-section is the following function of x: ______
$A(x)$= ______
Thus the volume of the solid is V = ______
This problem is similar to problems 29-34 of section 8.1 of the text.

7.(1 pt) The region between the graphs of $y = x^2$ and $y = 4x$ is rotated around the line $y = 16$.
The volume of the resulting solid is ______
This problem is similar to problems 26 and 27 of section 8.2 of the text.

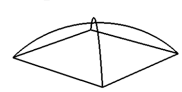
8.(1 pt) The region between the graphs of $y = x^2$ and $y = 6x$ is rotated around the line $x = 6$.
The volume of the resulting solid is ______
This problem is similar to problems 26 and 27 of section 8.2 of the text.

9.(1 pt) As viewed from above, a swimming pool has the shape of the ellipse

$$\frac{x^2}{6400} + \frac{y^2}{2500} = 1$$

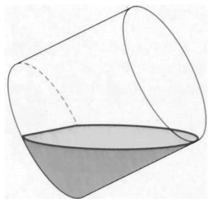
The cross sections perpendicular to the ground and parallel to the y-axis are squares. Find the total volume of the pool. (Assume the units of length and area are feet and square feet respectively. Do not put units in your answer.)
V = ______
This is similar to problem 47 in section 8.1 of the text.

10.(1 pt) The framework of a tent consists of a square base of side $54\sqrt{2}$ and two mutually perpendicular upright ribs formed from circular arcs of radius 90 joining diagonally opposite corners of the base. (Click on picture below for a better view.)



The tent fabric consists of four triangular flaps sewn together so as to fit snugly over the framework. Find the volume enclosed by the tent.
Answer: ______

11.(1 pt) A cylindrical bucket of radius 8 and height 23 full of water is tipped, and water pours out until the water coincides with a diameter of the base and just touches the rim of the bucket.



(Click on the picture above for a better view.) What is the volume of water left in the bucket?

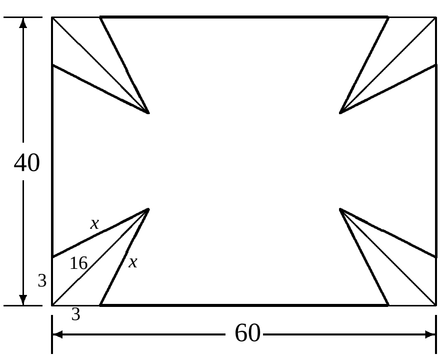
______

Hint: Compute volume by rectangular cross-sections parallel to the sides of the bucket.

12.(1 pt) A cylindrical bucket of radius 9 and height 23 containing some water is tipped, and begins to spill water when the angle of tilt is $\tan^{-1}(2)$. How much water was there in the bucket initially?

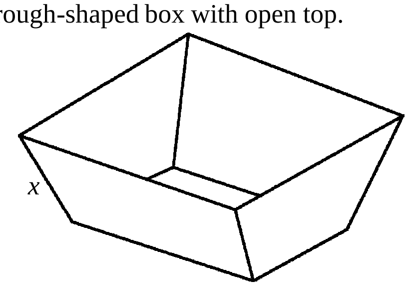
______

Hint: Compute volume by cross-sections parallel to the sides of the bucket and perpendicular to the water surface.



13.(1 pt)

Four equal-size kite-shaped wedges are cut from the corners of a 60 by 40 inch rectangle. The short sides of the wedges are 3 inches long, and the (long) diagonals are 16 inches. The sides of the cut rectangle are then folded up and glued along the long sides of the wedges (labelled x in the picture above) to form a trough-shaped box with open top.



What is the volume of the trough-shaped box?

We first compute that

$x =$ ______

The top of the trough is a rectangle of size ____ (long side) by ______

The bottom of the trough is a rectangle of size ____ (long side) by ______

Next noting that the horizontal cross-sections of the trough-shaped box are rectangles, we obtain that the volume is given by

$$V = \int_0^h A(y)\,dy$$

where h is the total (vertical) height of the trough-shaped box, and $A(y)$ denotes the area of the rectangular horizontal cross-section at height y (measured from the bottom of the box).

We find that $h =$ ______

and that $A(y) =$ ______

Thus the volume of the trough is $V =$ ______

14.(1 pt) A soda glass has the shape of the surface generated by revolving the graph of $y = 6x^2$ for $0 \le x \le 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of $1/2$ cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second. Do not put units in your answer. Also your answer should be positive, since we are asking for the rate at which the level DROPS rather than rises.)

answer: ______

This is similar to problem 52 in section 8.1 of the text.

15.(1 pt) Find the length of the curve defined by

$$y = 2\ln((x/2)^2 - 1)$$

from $x = 3$ to $x = 5$.

______

This is similar to problem 6 of section 8.3 of the text.

16.(1 pt) Find the length of the arc formed by

$$y = \frac{1}{8}\left(2x^2 - 4\ln(x)\right)$$

from $x = 2$ to $x = 8$

______

This is similar to problem 3 of section 8.3 of the text.

17.(1 pt) Find the length of the the graph of

$$y = \sqrt{16x + 32}$$

from $x = -2$ to $x = 14$.

Note: You can get full credit for this problem by just entering the final answer (to the last question) correctly. The initial questions are meant as hints towards the final answer and also allow you the opportunity to get partial credit.

The length of the graph is given by the integral $\int_a^b f(x)\,dx$, where $a =$ ______ , $b =$ ______ and $f(x) =$ ______

To simplify this integral we first need to substitute $u = \sqrt{g(x)}$, where

$g(x) =$ ______

To carry out this substitution we need to eliminate x from the integral by solving for x in terms of u. We obtain:

$x =$ ______

After carrying out this substitution, we obtain the definite integral $\int_c^d h(u)\,du$ where
$c =$ ______ , $d =$ ______ and
$h(u) =$ ______
Note: Use INF to denote ∞ and MINF to denote $-\infty$
After evaluating this integral, we obtain that the length of this graph is: ______

You will need to use the methods of section 7.4

18.(1 pt) Find the area of the surface obtained by rotating the graph of

$$y = \sqrt{25x+100}$$

from $x = -4$ to $x = 21$ about the x-axis.

______

This is similar to problem 4 of section 8.4 of the text.

19.(1 pt) Find the area of the surface obtained by rotating the graph of

$$y = \sin(8x)$$

from $x = 0$ to $x = \pi/8$ about the y-axis.

______

This is similar to problem 7 of section 8.4 of the text. You will need to use the integration methods of sections 7.3 and 7.2.