

PRACTICE1

Homework Set Hmwk4 due 5/2/09 at 6:00 AM

This set covers sections 6.6-6.8 of the text.

You may need to give 4 or 5 significant digits for some (floating point) numerical answers in order to have them accepted by the computer.

1.(1 pt) Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow 1} \frac{2^x - 2}{x^2 - 1} = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow \infty} \frac{\tan^{-1}(x)}{(1/x) - 2} = \underline{\hspace{2cm}}$$

Note: The first part is similar to problems 25 and 26 in Section 6.6 of the text.

2.(1 pt) Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow 0} \frac{1 - \cos(6x)}{1 - \cos(5x)} = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 1} \frac{6^x - 5^x - 1}{x^2 - 1} = \underline{\hspace{2cm}}$$

Note these are similar to problems 19, respectively 26, in Section 6.6 of the text.

3.(1 pt) Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow \infty} \frac{\ln(x^2 - 9)}{\ln(x) \cos(1/x)} = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow \infty} \frac{e^{9x}}{e^{10x} - e^{-10x}} = \underline{\hspace{2cm}}$$

4.(1 pt) Compute the following limits using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow 0} \frac{x}{\int_x^2 \sqrt{512 - 6t^3} dt} = \underline{\hspace{2cm}}$$

$$\lim_{x \rightarrow 0^+} \sin(x) \ln(x) = \underline{\hspace{2cm}}$$

Note the first question has something to do with the Fundamental Theorem of Calculus, whereas the second is similar to problem 35 in Section 6.6 of the text.

5.(1 pt) Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow \infty} \left(1 - \frac{3}{x}\right)^x = \underline{\hspace{2cm}}$$

Note that this question is similar to problems 46 and 47 in Section 6.6 of the text. See also Example 9 on page 407.

6.(1 pt) Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow \infty} (\sqrt[3]{x^3 - 7x^2} - x) = \underline{\hspace{2cm}}$$

7.(1 pt) Compute the following limit using l'Hôpital's rule if appropriate. Use INF to denote ∞ and MINF to denote $-\infty$.

$$\lim_{x \rightarrow 0^+} (\sin(x))^{\frac{1}{\ln(\sqrt{x})}} = \underline{\hspace{2cm}}$$

8.(1 pt) For each of the following forms determine whether the following limit type is indeterminate, always has a fixed finite value, or never has a fixed finite value. In the first case answer IND, in the second case enter the numerical value, and in the third case answer DNE. For example

$$\underline{\text{IND}} \quad \frac{0}{0}$$

$$\underline{0} \quad \frac{0}{1}$$

$$\underline{\text{DNE}} \quad \frac{1}{0}$$

To discourage blind guessing, this problem is graded on the following scale

0-9 correct = 0

10-13 correct = .3

14-16 correct = .5

17-19 correct = .7

Note that l'Hôpital's rule (in some form) may ONLY be applied to indeterminate forms.

— 1. $1^{-\infty}$

— 2. 0^{∞}

— 3. $\frac{0}{0}$

— 4. ∞^1

— 5. $\infty^{-\infty}$

— 6. 1^{∞}

— 7. ∞^{-e}

— 8. π^{∞}

— 9. $0 \cdot \infty$

— 10. $\frac{1}{-\infty}$

— 11. ∞^0

— 12. $0^{-\infty}$

— 13. $1 \cdot \infty$

— 14. 1^0

— 15. ∞^{∞}

— 16. $\frac{\infty}{0}$

— 17. 0^0

— 18. $\infty \cdot \infty$

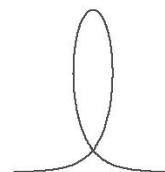
— 19. $\infty - \infty$

— 20. $\pi^{-\infty}$

9.(1 pt) The graph of the equation

$$y^3 + 18yx^2 = 2y^2 - 6x^2$$

has a self-intersection



at the point (_____ , _____).

Then the angle in degrees between the two tangent lines to the graph at this point is _____ .

(Hints: Use implicit differentiation to find $\frac{dy}{dx}$ as an expression in terms of x and y . Then find the point where this expression is indeterminate. Then apply l'Hopital's rule. Note also that there are two angles which might be called the angle between the tangent lines. We are looking for the smaller of the two)

10.(1 pt) Match the following differential equations with their solutions.

The symbols A , B , C in the solutions stand for arbitrary constants.

You must get all of the answers correct to receive credit.

- 1. $\frac{d^2y}{dx^2} + 25y = 0$
- 2. $\frac{dy}{dx} = \frac{-2xy}{x^2 - 5y^2}$
- 3. $\frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0$
- 4. $\frac{dy}{dx} = 10xy$
- 5. $\frac{dy}{dx} + 18x^2y = 18x^2$

- A. $y = Ae^{5x^2}$
- B. $y = Ce^{-6x^3} + 1$
- C. $y = A \cos(5x) + B \sin(5x)$
- D. $3yx^2 - 5y^3 = C$
- E. $y = Ae^{-6x} + Bxe^{-6x}$

This is similar to problems 1-8 in Section 6.7 of the text.

11.(1 pt) Find the particular solution of the differential equation

$$\frac{dy}{dx} = (x - 4)e^{-2y}$$

satisfying the initial condition $y(4) = \ln(4)$.

Answer: $y =$ _____

Your answer should be a function of x .

Note that this is similar to problem 7 in Section 6.8 of the text.

12.(1 pt) Find the particular solution of the differential equation

$$\frac{x^2}{y^2 - 8} \frac{dy}{dx} = \frac{1}{2y}$$

satisfying the initial condition $y(1) = \sqrt{9}$.

Answer: $y =$ _____

Your answer should be a function of x .

Note this is similar to problem 2 in Section 6.8 of the text.

13.(1 pt) Find the particular solution of the differential equation

$$\frac{dy}{dx} + 4y = 3$$

satisfying the initial condition $y(0) = 0$.

Answer: $y =$ _____

Your answer should be a function of x .

This is similar to problem 13 in Section 6.8 of the text.

14.(1 pt) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \cos(x) = 2 \cos(x)$$

satisfying the initial condition $y(0) = 4$.

Answer: $y =$ _____

Your answer should be a function of x .

This is similar to problem 16 in Section 6.8 of the text.