March 21, 2003 **Name** ____________________

Math 112.01 – Quiz 6 and Solutions

1) Find the particular solution of the initial value problem: $\begin{cases} \frac{x^2}{y^2-3}\frac{dy}{dx} = \frac{1}{2y} \\ y(1) = 2 \end{cases}$ Find y as an explicit function of x.

Solution: Separating the variables we have $\frac{2ydy}{y^2-3} = \frac{dx}{x^2}$. Integrating (use the substitution $u = y^2 - 3$ on LHS)

$\ln|y^2 - 3| = \frac{-1}{x} + C$

$y^2 = 3 + Ce^{\frac{-1}{x}}$

Using the initial condition, $C = e$ and we should take the positive square root. So the solution is $y = 3 + \sqrt{e^{1-\frac{1}{x}}}$

2) Compute the following limits: $i)\ \lim_{x\to\infty} \frac{-\ln(x)}{\sin(\frac{1}{x})}$ $ii)\ \lim_{x\to\infty} (1 - \frac{1}{x})^x$

Solution:

i) Note as $x \to \infty$, $\ln(x) \to \infty$ and $\sin(\frac{1}{x}) \to 0^+$. Therefore this limit is $-\infty$. LR CANNOT be applied here.

ii) This is an instance of the indeterminate form 1^∞.

$\lim_{x\to\infty} (1 - \frac{1}{x})^x = \lim_{x\to\infty} e^{x\ln(1-\frac{1}{x})}$

Now we need to determine, $\lim_{x\to\infty} x\ln(1 - \frac{1}{x}) = \lim_{x\to\infty} \frac{\ln(1-\frac{1}{x})}{\frac{1}{x}} = \lim_{x\to\infty} \frac{\frac{1}{(1-\frac{1}{x})} \cdot \frac{1}{x^2}}{\frac{-1}{x^2}} = -1$ by LR

Therefore, the final answer is $e^{-1} = \frac{1}{e}$