May 17 , 2001 **Name** ______________________________

Some Practice Problems for the Final Exam

Caution: These problems may not cover everything. For instance, they don't cover sequences and series which will be on the final.

1) Consider the definite integral $\int_{\frac{-1}{\sqrt{2}}}^{0}(x+\sqrt{1-x^2})dx$. This integral represents the area of a certain region. Rewriting the integrand as $x-(-\sqrt{1-x^2})$, interpret this as an area between two graphs (upper curve - lower curve).

a) Draw the region whose area is represented by this integral. Then find the area of the region by an elementary formula, without performing an integration.

b) Directly evaluate the integral $\int_{\frac{-1}{\sqrt{2}}}^{0}(x+\sqrt{1-x^2})dx$ (using an appropriate technique of integration) to find the area of the region above.

2) Find the area of the region enclosed by $x=y^2$ and $y=2-x$. (Sketch the region first)

3) Evaluate the following limits

$$i)\ \lim_{x\to 0}\frac{\int_0^x \arccos t \sin t dt}{x^2} \qquad ii)\ \lim_{x\to 0^+}(x+e^x)^{1/x} \qquad iii)\ \lim_{x\to\infty}\frac{e^{-x}}{\ln x}$$

4) Find the particular solution of the differential equation

$$\frac{dx}{dy}=\frac{x^2-1}{2(y^2+1)}$$

satisfying the condition $y(0)=\frac{\pi}{4}$

5) Evaluate each of the following limits. In case there is an indeterminacy, state the form of the indeterminacy first.

$$i)\ \lim_{x\to 1^+}x^{\frac{1}{x-1}} \qquad ii)\ \lim_{x\to\infty}x\cdot\sin(\frac{\pi}{x}) \qquad iii)\ \lim_{x\to 0}\frac{\arcsin(x)}{\cos(x)} \qquad iv)\ \lim_{x\to-\infty}\frac{x^8-x^5+3x^2+10^9}{2x-x^4-x^6-3x^8} \qquad v)\ \lim_{x\to\pi}\frac{\cos(x)}{\sin(x)}$$

6) State why the integral

$$\int_1^2\frac{dx}{x\ln(x)}$$

is improper and determine whether it converges. If it does, find its value.

7) The following sum

$$\sqrt[3]{1+\frac{7}{n}}\,\frac{7}{n}+\sqrt[3]{1+\frac{14}{n}}\,\frac{7}{n}+\sqrt[3]{1+\frac{21}{n}}\,\frac{7}{n}+\cdots+\sqrt[3]{1+\frac{7n}{n}}\,\frac{7}{n}$$

is a right Riemann sum for a certain definite integral. Using this information compute the limit of this sum as $n \to \infty$.

8) The base of a certain solid is the region bounded from below by the parabola $y = 5x^2$ and from above by the line $y = x$. Cross-sections perpendicular to the x-axis are squares. Find the volume of the solid. (Note: This is **not** a solid of revolution.)

9) Find the indicated integrals (if they exist).

(a) $\displaystyle\int_{-2/3}^{2/3} x\sqrt{3x+2}\,dx$

(b) $\displaystyle\int \sin^5(2x)\cos^4(2x)\,dx$

(c) $\displaystyle\int_0^1 2x\ln(x+2)\,dx$

(d) $\displaystyle\int \frac{1}{x^2-4x+3}\,dx$

10) (25 points) Find length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}, \quad -2 \le x \le -1$$