Exam 1

Math 112: Calculus B Name: ____________________

100 points 02/07/2001

• You must show all work to receive full credit.

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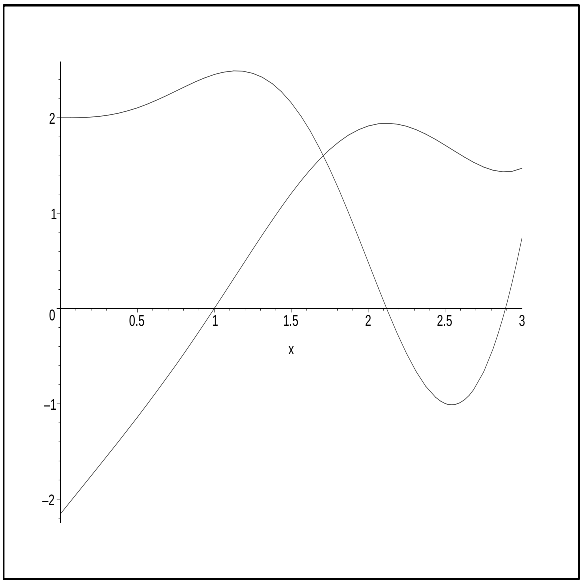
1) (8 pts) The graph of two functions are shown below. One is the graph of a function $f(x)$ the other is a graph of its area function $A_f(x)$. Determine which is $f(x)$ and which is $A_f(x)$. Explain how you arrived at your conclusion.



Solution: Since $A_f(x)$ is an antiderivative of $f(x)$ then it follows that $f(x)$ is the derivative of $A_f(x)$. From differential Calculus we know that $f(x)$ should be positive when $A_f(x)$ is increasing and $f(x)$ should be negative when $A_f(x)$ is decreasing. Further, $f(x)$ must be zero when $A_f(x)$ attains an extreme value. From these observations the function with initial point at approximately (0, -2.1) and terminal point at approximately (3, 1.5) is $A_f(x)$.

2) (8pts) Describe what the mean value theorem says and how it relates to the average value of a function.

Solution: The mean value theorem for integrals states that if f is a continuous function on $[a, b]$ then there is a point, c, in $[a, b]$ such that $f(c) = \dfrac{\int_a^b f(x)\,dx}{b-a}$.

The average value of f on the interval $[a, b]$ is given by $f(c)$.

3) (24 pts) Given that $A_f(x) = \int_0^x f(t)\,dt$ write each of the following in terms of the area function.

a) $\int_{-4}^{1} \frac{\mathrm{f}(t)}{\sqrt{2}}\,dt$

$$\begin{aligned}
\int_{-4}^{1} \frac{\mathrm{f}(t)}{\sqrt{2}}\,dt &= \frac{1}{\sqrt{2}}\{\int_{-4}^{0} \frac{\mathrm{f}(t)}{\sqrt{2}}\,dt + \int_{0}^{1} \frac{\mathrm{f}(t)}{\sqrt{2}}\,dt\} \\
&= \frac{1}{\sqrt{2}}\{-\int_{0}^{-4} \frac{\mathrm{f}(t)}{\sqrt{2}}\,dt + \int_{0}^{1} \frac{\mathrm{f}(t)}{\sqrt{2}}\,dt\} \\
&= \frac{1}{\sqrt{2}}\{-A_f(-4) + A_f(1)\}
\end{aligned}$$

b) $\int_{-3}^{3} f(t)\,dt$

$$\begin{aligned}
\int_{-3}^{3} f(t)\,dt &= \int_{-3}^{0} f(t)\,dt + \int_{0}^{3} f(t)\,dt \\
&= -\int_{0}^{-3} f(t)\,dt + \int_{0}^{3} f(t)\,dt \\
&= -A_f(-3) + A_f(3)
\end{aligned}$$

c) $\int_{6}^{2x} f(t)\,dt$

$$\begin{aligned}
\int_{6}^{2x} f(t)\,dt &= \int_{6}^{0} f(t)\,dt + \int_{0}^{2x} f(t)\,dt \\
&= -\int_{0}^{6} f(t)\,dt + \int_{0}^{2x} f(t)\,dt \\
&= -A_f(6) + A_f(2x)
\end{aligned}$$

4) (36 pts) Evaluate the following using the Fundamental Theorem of Calculus.

a) $\int_2^4 \{2\ \sin(x) - 4x\}\, dx$

$$\begin{aligned}\int_2^4 \{2\sin(x) - 4x\}\, dx &= -\cos(x) - 2x^2|_2^4 \\ &= [-2\cos(4) - 32] - [-2\cos(2) - 8] \\ &\approx -23.52\end{aligned}$$

b) $\int_1^2 \{\frac{1}{x^3}\}\, dx$

$$\begin{aligned}\int_1^2 \{\frac{1}{x^3}\}\, dx &= \frac{-1}{2x^2}|_1^2 \\ &= \frac{-1}{8} - -12 \\ &= 0.375\end{aligned}$$

c) $\frac{d}{dt}\int_4^t \ln(xe^x)\, dx$

$$\frac{d}{dt}\int_4^t \ln(xe^x)\, dx = \ln(te^t)$$

d) $\int_2^x \frac{d}{dt}\{\cos(\ln(t))\}\, dt$

$$\begin{aligned}\int_2^x \frac{d}{dt}\{\cos(\ln(t))\}\, dt &= \cos(\ln(t))|_2^x \\ &= \cos(\ln(x)) - \cos(\ln(2))\end{aligned}$$

5) (8pts) Use the Fundamental Theorem of Calculus to find $f(x)$ provided $A_f(x) = \{\sin(2x-1)\}^2$.

$$\begin{aligned} f(x) &= \frac{d}{dx} A_f(x) \\ &= 2\sin(2x-1)\cos(2x-1)2 \\ &= 4\sin(2x-1)\cos(2x-1) \end{aligned}$$

6) (8 pts) Use the Fundamental Theorem of Calculus to find

$$\frac{d}{dt}\int_{2t}^{3t^2} \ln(x)\,dx$$

$$\begin{aligned} \int_{2t}^{3t^2} \ln(x)\,dx &= \int_{2t}^{0} \ln(x)\,dx + \int_{0}^{3t^2} \ln(x)\,dx \\ &= -\int_{0}^{2t} \ln(x)\,dx + \int_{0}^{3t^2} \ln(x)\,dx \end{aligned}$$

$\int_{2t}^{3t^2} \ln(x)\,dx = -A_f(2t) + A_f(3t^2)$ and $\frac{d}{dt}\{-A_f(2t) + A_f(3t^2)\} = -2f(2t) + 6tf(3t^2)$

thus

$$\frac{d}{dt}\int_{2t}^{3t^2} \ln(x)\,dx = -2\ln(2t) + 6t\ln(3t^2)$$

7) Estimate the area under the curve $f(x) = \frac{1}{\sqrt{1-x^3}}$ on the interval [-2,0] using four approximating rectangles with height defined using right endpoints.

$$a = -2, b = 0, n = 4, \Delta x = \frac{0-(-2)}{4} = \frac{1}{2}$$

$$x_i = -2 + i(\frac{1}{2})$$

$$\begin{aligned} Area &= \sum_{i=1}^{4} f(x_i)\Delta x \\ &= \sum_{i=1}^{4} f(-2 + (\frac{i}{2}))\frac{1}{2} \\ &= \frac{1}{2}\{f(\frac{-3}{2}) + f(-1) + f(\frac{-1}{2}) + f(0)\} \\ &\approx \frac{1}{2}\{0.47809 + 0.70711 + 0.942809 + 1\} \\ &\approx 1.564 \end{aligned}$$