

Problem 3: Weights of Vectors

Let $\mathbb{Z}_n^m = \{(x_1, x_2, \dots, x_m) : x_i \in \mathbb{Z}_n\}$ be the set of all vectors of length m with entries from $\mathbb{Z}_n = \{0, 1, \dots, n-1\}$, integers $\pmod n$.

1. What is the size of the set \mathbb{Z}_n^m ?
2. The weight of a vector $\mathbf{x} = (x_1, x_2, \dots, x_m) \in \mathbb{Z}_n^m$ is the number of non-zero components of \mathbf{x} . For example, the weight of the binary vector $(1, 0, 0, 1, 0, 0, 1)$ is 3. If we pick a random vector from \mathbb{Z}_3^{10} , what is the probability that its weight is 4? If you pick 100 such vectors randomly, how many do you expect to get with weight 4?

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 1/29/2022. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon Saturday, Feb 5, 2022. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.