Problem of the Week-1: Maximum Value of a Function

The function $f(x) = \sqrt{x} - r - \frac{x-r^2}{2r+1}$ was important in a medieval algorithm to approximate square roots of integers. Find the maximum value of this function over the interval $[r^2, (r+1)^2]$, where r is an integer and show that this maximum goes to 0 as $r \to \infty$. What is the geometric meaning of this fact?

As always, show your work, fully explain and justify your answer.

Posting Date 1/10/15. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 1/23/15.