

Problem of the Week-4: An Algorithm for Approximate Square Roots

Medieval Muslim mathematicians (e.g. Kushyar, al-Kashi) devised algorithms to find approximate square roots and higher order roots of integers (e.g. fifth roots). In one of those algorithms to find an approximate square root of a 6-digit number $N = abcdef$ as a rational number, they knew that the integer part of the approximate root will be a 3 digit integer $r = ABC$ such that $r^2 \leq N$.

1. The first step of the algorithm is to find the largest digit A such that $A^2 \leq ab$. Show that this is the correct first digit of the integer part of the approximation.
2. In finding the fractional part of the approximation, they assumed that the square root function is linear. Under this assumption find the numerator and denominator of the fractional part in terms of N and r .

As always, explain and justify your answer.

Posting Date 2/23/11. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 3/4/11.