Problem 4: Power of Counting

Give a counting argument to find the value of the following sum (find a closed formula)

$$\sum_{k=1}^{n} \binom{n}{k} k^3$$

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 10/6/2023. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by noon on Friday, October 20, 2023. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.