

Problem 5: Matrices over Finite Fields

Let \mathbb{F}_q be the finite field with q elements, and let V be an n -dimensional vector space over \mathbb{F}_q

1. Determine the number of elements in V .
2. Let $GL_n(\mathbb{F}_q)$ denote the set of $n \times n$ invertible matrices over \mathbb{F}_q (called the general linear group). Find the size (number of elements) of $GL_n(\mathbb{F}_q)$.
3. Let $SL_n(\mathbb{F}_q)$ be the subset (and subgroup) of $GL_n(\mathbb{F}_q)$ consisting of matrices with determinant 1 (called the special linear group). Find the size of $GL_n(\mathbb{F}_q)$.

As always, show your work, fully explain and justify your answer. A solution mainly obtained by computers or calculators will not be accepted.

Posting Date 10/10/2020. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 by e-mail or hard-copy by 4 pm on Oct 23, 2020. An email submission must be a single pdf file. Hard copy submissions must be dropped in the file holder at my office door (Hayes 319) and must include a time stamp.