Problem 7: Fibonacci Sequence mod N

Let $N > 3$ be a positive integer and consider the Fibonacci sequence mod N, that is,

$$a_1 = a_2 = 1, \text{ and } a_{n+1} = (a_n + a_{n-1}) \mod N \text{ for } n \geq 2.$$

Prove that the sequence $a_1, a_2, a_3, \ldots$ is periodic. What is the maximum possible length of the period?

As always, show your work, fully explain and justify your answer.

Posting Date 11/27/16. Submit solutions to Noah Aydin, Mathematics Department, RBH 319 (e-mail or hard-copy, but hard copy submissions must include a time stamp) by 4 pm on 12/8/16.