Problem of the Week-6: Number of Prime Factors

Show that for every positive integer n, the number $2^{2^n} + 2^{2^{n-1}} + 1$ has at least n distinct prime factors.

As always, show your work, fully explain and justify your answer.

Hint: The factorization $x^4 + x^2 + 1 = (x^2 - x + 1)(x^2 + x + 1)$ may be useful.